INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date———FN/AN 2 Hrs. Full Marks: 30 No. of Students: 65

Mid-Autumn Semester 2018-2019 Deptt: MATHEMATICS Sub No: MA 31005

——Yr. B.Tech.(H)/B.Arch.(H)/M.Sc. Sub. Name: Real Analysis

Instruction: Answer all questions, which are of equal values

1. (a) Let A be any set of real numbers. Show that if u is the lub of A, then for every $\epsilon > 0$, there exists $a \in A$ such that $u - \epsilon < a \leq u$.

   (b) State and prove the Archimedean Principle.

2. (a) Define an algebraic number and show that set of algebraic numbers is countable.

   (b) Let $S = \{x | x = \frac{n}{n+1}, n \in \mathbb{N}\}$. Show that $lub\, S = 1$.

3. (a) Show that a set G is open if and only if its complement is closed.

   (b) If $\{I_n\}$ is a sequence of closed and bounded intervals in $\mathbb{R}$, such that $I_n \supset I_{n+1}$ $(n = 1, 2, \cdots)$, then show that $\bigcap_1^\infty I_n$ is non empty.

4. (a) Show that compact subsets of metric spaces are closed.

   (b) Give an example of an open covering of the open interval $(0, 1)$ which has no finite sub cover. Is $(0, 1)$ compact? Justify your answer.

5. (a) Show that every bounded infinite subset of $\mathbb{R}^k$ has a limit point in $\mathbb{R}^k$.

   (b) Let (X, d) be any metric space, then show that (X, d^*) is a metric space with the metric $d^*(x, y) = \frac{d(x,y)}{1+d(x,y)}$, $\forall x, y \in X$.

6. (a) State and prove the Heine Borel Theorem.

   (b) Construct a bounded set of real numbers with exactly 5 limit points.