INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

AEROSPACE ENGINEERING DEPARTMENT

Date: Time: 2 Hrs, Full Marks: 20

No of Students: 59 Mid-Autumn Semester Examination

AE21001 **Introduction to Aerodynamics**

2nd Year B. Tech. (H) & DD

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*Answer **any four** full questions. All parts of a multi-part question must be answered together.*

Assumptions, if required, can be made with appropriate justifications.

Some important vector identities:

$$\nabla\times(s\vec{u})=s\nabla\times\vec{u}-\vec{u}\nabla s$$

$$\nabla\times(\vec{u}\times\vec{v})=\vec{v}\nabla\vec{u}+\vec{u}\nabla\cdot\vec{v}-\vec{u}\nabla\vec{v}-\vec{v}\nabla\cdot\vec{u}$$

$$\nabla^2\vec{u}=\nabla\cdot\nabla\vec{u}=\nabla(\nabla\cdot\vec{u})-\nabla\times(\nabla\times\vec{u})$$

$$\vec{u}\nabla\vec{u}=\frac{1}{2}\nabla(\vec{u}\cdot\vec{u})+(\nabla\times\vec{u})\times\vec{u}$$

$$(\vec{u}\cdot\nabla)\vec{u}=\frac{1}{2}\nabla(\vec{u}\cdot\vec{u})-\vec{u}\times\nabla\times\vec{u}$$

Notations have their usual meaning unless specified otherwise.

1(a) Show that the stress tensor in a fluid is symmetric.

(b) Considering the Eulerian velocity at two neighbouring positions in space show that the fluid velocity is the superposition of a uniform velocity, an isotropic expansion, a pure straining motion without change in volume and a rigid body rotation.

(c) Show that in an incompressible flow with small or negligible temperature changes the velocity field is solenoidal.

2(a) Define a vortex line. The Eulerian velocity components in a fluid motion are given by $u=a+by-cz$, $v=d-bx+ez$ and $w=f+cx-ey$ where a, b, c, d, e, f are constant. Find the equation of the vortex lines.

(b) Show that a vortex filament cannot end within a fluid.

(c) Compute the velocity difference across a surface vortex of strength Γ.

3(a) Define singly-connected and multiply-connected domains. How many irreconcilable circuits can be drawn in doubly-connected region? The velocity potential that describes solenoidal and irrotational fluid motion in a doubly connected region of space is many valued – why? What conditions are required to be satisfied so that the solenoidal velocity

field in a doubly connected region can be determined uniquely? Are these conditions required if the flow is in a singly connected domain?

(b) Show that in incompressible and inviscid flow the rate of change of circulation around a material closed curve is zero if the body force field is conservative.

4(a) Obtain an equation for the rate of change of internal energy of an element of fluid and show that the work done by the viscous stresses represents one-way transfer of energy.

(b) A jet of water 20 mm in diameter strikes a flat plate at an angle of 30^0 to the normal of the plate. The jet velocity is 10 m/s and uniform over the cross section just before impact. Find the force exerted by the jet on the plate when the plate is stationary. What will be the force if the plate moves against the jet with a velocity of 2 m/s? Viscous stresses can be ignored.

(c) Define and explain the chord of an airfoil and camber of an airfoil.

5(a) Assuming incompressible flow of a constant property fluid show that the local rate of change of vorticity in a flow is given by the sum total of vorticity convected with the flow, vorticity diffused by the viscous action and local redistribution of vorticity due to rotation and stretching.

(b) Water is steadily running down the upper face of a very wide plane surface inclined at an angle θ to the vertical in a layer of uniform thickness. Find the thickness of the layer in terms of the volume flux and the inclination. Draw a neat sketch of the velocity profile.