INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR:
MID-SEMESTER EXAM - 2012 AUTUMN

NETWORK THEORY (EC 21005). FM: 100 2HRS.

Guidelines:- i) Answer all 5 questions.
ii) Make necessary assumptions, with justifications.
iii) Attempt all parts of a question in one place. Box/Encircle the final answer.

Q-1. If the Laplace Transform of $f(t)$ is $F(s)$.

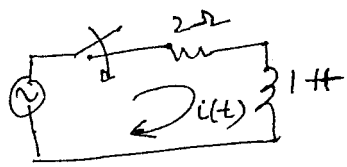
a) Derive the Laplace transform of $\{t f(\alpha t)\}$ in terms of $F(s)$. Note 'α' is a constant related to scaling. [5]

b) Derive the Laplace Tx. of $\{t^{-1} f(t)\}$ in terms of $F(s)$. Using the above result, evaluate the integral, $\int_0^\infty \frac{\sin t}{t} e^{-t}\, dt$. [10]

Q-2. Consider the signal, $f(t) = \frac{1}{\sqrt{t}} u(t)$, where $u(t)$ is the unit step function. We define $g(t) \triangleq f(t) * f(t)$, where '$*$' is the convolution operator. Using the result of $g(t)$, derive the Laplace Tx. of $f(t)$. [15]

[Hint:- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$].

Q-3. Consider the R-L ckt. [Note:- You cannot use Laplace Tx.]

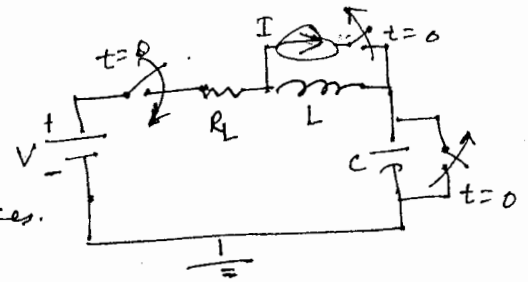


(a) Write down the Differential Eqn. relating $i(t)$ & $v(t)$. [5]

(b) Find the Sol^n. to the homogeneous eqn. [5]

(c) If $v(t) = e^{-2t}(1 + t^2 \sin \omega t)$. Find the particular Sol^n. [10]

Q-4 System design Problem:-



Consider the following ckt:-

→ V & I are DC voltage & current sources.

→ Load resistance, $R_L = 25\Omega$

→ Capacitor = 50 pF.

→ Inductor = 10 nH, with a Quality (Q) factor of $2\sqrt{2}$, at the resonant frequency of the L-C tank network.

a) Write down the integro-differential eqn, that $i(t)$ satisfies for $t \geq 0$. [Note: that the inductor is not ideal]. [5]

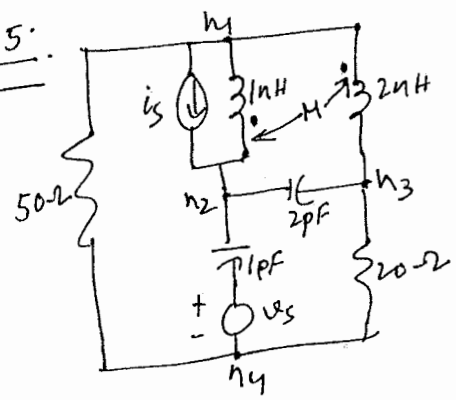
b) Find the Soln. to the Differential Eqn. along with initial conditions. [5]

c) Consider that $i(t)$ reaches a peak at $t = (\ln 2)$ ns. Sketch the waveform by finding the appropriate value of V & I. Find the pulse repetition frequency (PRF)*. [12.5]

d) Consider now that $i(t)$ reaches a peak at $t = (2\ln 2)$ ns. Sketch the corresponding waveform & find the PRF.* [12.5]

*NOTE:- PRF is related to inverse of minimum time period. The minimum time period consists of the time reqd. for the pulse to decay to 5% of its peak value and the charging time (of the current through the inductor to the appropriate initial value).

Q-5.



n_1, n_2, n_3, n_4 are the nodes.
Mutual inductance = 0.5 nH.
Freq = 100 MHz, $v_s = 0.25 \sin \omega t$
$i_s = 0.3 \cos \omega t$.

a) Write down the Incidence ~~Admittance~~ matrix, Fundamental loop matrix & Impedance matrix. [5]

b) Derive the matrix formulation of the loop currents and the corresponding matrix elements. [10]