AGRICULTURAL AND FOOD ENGINEERING DEPARTMENT
INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date of Examination: 18.02.2014 (AN) **Time: 2 h**
Mid-Spring Semester, 2014 **Full Marks: 30**
M.Tech. in LWRE
Sub. No. and Name: AG60042; Water Resources Systems Analysis
No. of Students: 15 (M.Tech. LWRE) + 1 (MS Civil)

Instructions: All questions are compulsory. Make reasonable assumptions, if necessary.

Q1. The agricultural output of three farms A, B and C is limited by both the amount of available irrigable land and the quantity of water allocated for irrigation as shown in the Table below.

Farm	Usable Land (Acres)	Water Allocation (Acre Feet)
A	400	600
B	600	800
C	300	375

Three crops i.e. sugar beet, cotton and sorghum are being considered for the coming season. These three crops differ primarily in their expected net return per acre and their consumption of water. In addition, the authorities have set a maximum quota for the total acreage that can be devoted to each of these crops. Details of these are shown in the table below.

Crop	Maximum Quota (Acres)	Water Consumption (Acre Feet/Acre)	Net Return (Rs/Acre)
Sugar beets	600	3	1000
Cotton	500	2	750
Sorghum	325	1	250

Because of limited water available for irrigation, the farms will not be able to use all of its irrigable land for planting crops in the coming season. To ensure equity between the three farms, it has been agreed that every farm will plant the same proportion of its available irrigable land. For example, if Farm A plants 200 of its available 400 acres, then Farm B must plant 300 of its available 600 acres and Farm C must plant 150 of its available 300 acres. However, any combination of the crops may be grown in any of the farms. Formulate a linear programming problem to maximize the total net returns from the three farms as a whole.

(6)

Q2. Solve the following LP using Graphical Method. Use of Graph Sheet is not necessary.

Maximize $Z = 3X_1 + 2X_2$

Subject to $-2X_1 + 3X_2 \leq 9,$

$3X_1 - 2X_2 \leq -20,$

$X_1, X_2 \geq 0$

(3)

Q3. Write the Dual of the given LP problem and solve it using Dual Simplex method:

Maximize $Z = 4X_1+3X_2$
Subject to $0 \le X_1 \le 6$
$0 \le X_2 \le 8$
$X_1 + X_2 \le 7$
$3X_1 + X_2 \le 15$
$-X_2 \le 1$

(6)

Q4. There are three reservoirs with daily supplies of 20, 25 and 15 million litres of fresh water, respectively. On each day water must be supplied to four cities A, B, C and D whose demands are 10, 12, 8 and 15 million litres, respectively. The cost of pumping per million litres is given below:

		Cities			
		A	B	C	D
Reservoirs	1	5	3	4	2
	2	2	3	5	2
	3	3	2	1	4

Determine the cheapest pumping schedule if the excess water can be disposed off at no cost.

(6)

Q5. The following tableau gives an optimal solution to a standard linear program:

Maximize: $Z = \mathbf{CX}$, Subject to: $\mathbf{AX} = \mathbf{B}, \mathbf{X} \ge 0$

C_j / Basis	2	3	1	0	0	
	X_1	X_2	X_3	X_4	X_5	B
X_1	1	0	-1	3	-1	1
X_2	0	1	2	-1	1	2
$\bar{C}$ Row	0	0	-3	-3	-1	Z=8

(a) How much can C_2 be varied without affecting the optimal solution? Find the optimum solution when $C_2 = 1$?

(b) Find the range on λ for which the given basis (X_1, X_2) is still optimal if the original **B** vector is replaced by $\mathbf{B} + \lambda\mathbf{B}^*$ where $\mathbf{B}^* = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$ and $-\infty < \lambda < \infty$.

Also, find the optimal solution when $\lambda = \frac{1}{2}$. (Assume that (X_4, X_5) formed the initial basis). Further, find the optimal solution when $\lambda = 2$.

(c) Find the optimal solution when a new constraint, $X_1 + X_2 \ge 2$ is added to the original problem.

(d) Compute the shadow prices of the constraints of the original problem.

(9)

GOOD LUCK!