INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date: FN/AN **End-Semester** Time: 3Hrs. Full Marks: 100 No. of students: 100
Spring Semester 2010-2011 Dept. Electronics & ECE Sub. No. EC21004
2^{nd} Year B.Tech.. Sub. Name: Signals and Systems

Answer all questions. Answers should be brief, ***to the point and legible****. Sketches wherever appear should be neat and* ***properly labeled****. All parts of a question to be answered at one place.* ***All steps*** *towards a solution must be* ***unambiguously presented****.*

1.(a) Write the impulse response and transfer function of a distortion less system. Sketch the magnitude and phase response. How do you say that this is distortionless?

(b) Find the DTFT representations for the following periodic signal : Sketch the magnitude and phase spectra.

$$x(n) = \cos(\frac{\pi}{8}n) + \sin(\frac{\pi}{5}n)$$

(c) The input to a discrete-time system is given by

$$x(n) = \cos(\frac{\pi}{8}n) + \sin(\frac{3\pi}{4}n)$$

Use the Discrete Time Fourier Transform (DTFT) to find the output of the system, $y[n]$, for the following impulse responses $h[n]$

(i) $h(n) = \dfrac{\sin(\frac{\pi}{4}n)}{\pi n}$ (ii) $h(n) = (-1)^n \dfrac{\sin(\frac{\pi}{2}n)}{\pi n}$

(2+2+2)+(2+2+2)+(4+4)

2 (a) State Nyquist Theorem. Show the derivation of interpolation technique by which one can recover original time domain signal from sampled version theoretically.

(b) For the following signal sampled with sampling interval T_s, determine the bounds on T_s that will guarantee no aliasing.

$$x(t) = \frac{1}{t}\sin(3\pi t) + \cos(2\pi t)$$

(c) A continuous-time signal lies in the frequency band $|\omega| < 5\pi$. This signal is contaminated by a large sinusoidal signal of frequency 120π. The contaminated signal is sampled at a sampling rate of $\omega_s = 13\pi$. After sampling, at what frequency does the sinusoidal interfering signal appear?

(d) Is it possible to recover the original signal from samples obtained through undersampling? – Explain.

(2+4) + 4 +6 + 4

3. (a) Show how decimation-in-time algorithm reduces Discrete Fourier Transform (DFT) computation. Show the computation gain for 1024 point DFT.

(b) Compute DFT of sample sequence $\{x[n]\} = \{1, 0, -1, 1, 0, 1, -1, -1\}$ directly and through Fast Fourier Transform signal flow diagram.

(c) Derive the half-power bandwidth of the system with impulse response

$$h(n) = (\frac{7}{8})^n u(n)$$

(4+2) + (4+4) + 6

Q4. (a) Find z-transform of

$$x(n) = n((\frac{1}{2})^n u[n] * (\frac{1}{4})^n u[n-2])$$

Where * represents convolution.

(b) Use the following clues to determine the signal $x[n]$ and rational z-transform $X(z)$.

(i) $X(z)$ has poles at $z = 1/2$ and $z = -1$,
(ii) $x[1] = 1, x[-1] = 1$,
(iii) and the ROC includes the point $z = 3/4$.

(c) Use the unilateral z-transform to determine the forced response, the natural response, and the complete response of the systems described by the following difference equation with the given input and initial conditions.

$$y(n) - \frac{1}{3}y[n-1] = 2x[n], \quad y[-1] = 1, \quad x[n] = (\frac{-1}{2})^n u[n]$$

6 + 6 + (3+3+2)

5. (a) A continuous time system is characterized by $H(s) = 1/(s^2 + 40s + 300)$. Design a discrete time system that approximates continuous time system. Consider 100 Hz sampling rate and zero-order-hold. Sketch the output.

(b) Explain the Gram-Schmidt orthogonalization technique.

(c) Distinguish between Strict Sense Stationary and Wide Sense Stationary random process. Consider a random process, $X(t) = A\cos(\omega t + \theta)$ where θ is uniform random variable in the range $[-\pi, \pi]$ and A, ω are constant. Find if the random process is Wide Sense Stationary.

(d) Explain the principles of working of Weiner-Hopf filter.

– x –

(4+2) + 4 + (2+4) + 4