CS21004 Formal Languages and Automata Theory, Spring 2012–13

Mid-Semester Test

Maximum marks: 34 | Date: 22-Feb-2013 | Duration: Two hours

Roll no: _____ **Name:** _____

[*Write your answers in the question paper itself. Be brief and precise. Answer all questions.*]

1. Let $L_1 = \mathcal{L}(S)$ and $L_2 = \mathcal{L}(T)$, where the non-terminal symbols S and T satisfy the productions:

$$S \rightarrow a \mid b \mid abSS.$$
$$T \rightarrow a \mid b \mid TTab.$$

Find examples of strings α of length forty such that:

(a) $\alpha \in L_1$ and $\alpha \in L_2$. _____

(b) $\alpha \in L_1$ and $\alpha \notin L_2$. _____

(c) $\alpha \notin L_1$ and $\alpha \in L_2$. _____

(d) $\alpha \notin L_1$ and $\alpha \notin L_2$. _____

For each part, only one example of length forty suffices. **(4)**

2. Convert the following grammar (over the alphabet $\{a, b, c, d\}$) to the Chomsky normal form. **(6)**

$$S \rightarrow aSd \mid T,$$
$$T \rightarrow bTc \mid \epsilon.$$

3. Two strings α, β of the *same length* over the alphabet $\{0, 1\}$ are said to have *Hamming distance* k if they differ in exactly k positions. For example, the strings 1101110010 and 1101011010 have Hamming distance two, because they differ only in the fifth and the seventh positions. Let L be a language. By $H_k(L)$, we define the language consisting of strings α such that α is at Hamming distance k from some string in L. If L is regular, prove that $H_k(L)$ is regular for each fixed $k \geqslant 0$. **(6)**

(**Warning:** You are your friend's friend. So be careful.)

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4. A context-free grammar is called strongly *right linear* if each production in the grammar is of the form $A \rightarrow aB$ or $A \rightarrow \epsilon$, where A, B are non-terminal symbols and a is a terminal symbol. Prove that L is the language of a strongly right-linear grammar if and only if L is regular. **(6)**

5. One of the following two languages is context-free, and the other is not. Identify which one is what. Justify. **(6+6)**

(a) $L_a = \{a^l b^m c^n \mid l, m, n \geqslant 0,\ l + m \geqslant n\}$.

(b) $L_b = \{a^l b^m c^n \mid l, m, n \geqslant 0,\ l \geqslant n \text{ and } m \geqslant n\}$.