Department of Physics | Indian Institute of Technology Kharagpur
Sub: Quantum Mechanics-II (PH41010) | End-Spring Semester Examination
Full marks: 50 | Time - 3 hours

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1. Show that $(\sigma \cdot \pi)\ (\sigma \cdot \pi) = \pi^2 - e\sigma \cdot \mathbf{B}$, where $\pi = \mathbf{p} - e\mathbf{A}$ is the operator corresponding to kinetic momentum. $\mathbf{A}$ is the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$ and σ's are the Pauli matrices . [4]

2. Show that, under Lorentz transformation, the bilinear forms $\bar{\psi}\psi$ and $\bar{\psi}\gamma^\mu\psi$ transform like a scalar and a four vector respectively. [4]

4. Prove that the Dirac equation is covariant under a Lorenz transformation $\psi(x) \rightarrow \psi'(x') = S(\Lambda)\psi(x)$, provided $\Lambda^\mu{}_\nu \gamma^\nu = S^{-1}\gamma^\mu S$. [4]

4. Check that the Dirac gamma matrices (γ^μ) in Pauli-Dirac representation and in Majorana representation (γ^μ_M) are related by a unitary transformation $\gamma^\mu_M = S\gamma^\mu S^{-1}$, with $S = \frac{1}{\sqrt{2}}\gamma^0(1 + \gamma^2)$. [5]

5. Calculate [4 +4 +4]
(i) $\mathrm{Tr}\,(\not{a}_1\not{a}_2\not{a}_3\not{a}_4\not{a}_5\not{a}_6)$
(ii) $\mathrm{Tr}\left[(\not{p} - m)\gamma^\mu(1 - \gamma_5)(\not{q} + m)\gamma^\nu\right]$
(iii) $\gamma^\mu(1 - \gamma_5)(\not{p} - m)\gamma_\mu$

6. Consider the Dirac equation in one dimension, $H\psi = i\hbar\frac{\partial\psi}{\partial t}$, where the Hamiltonian $H = \alpha p_z + \beta m + V(z)$, with $\alpha = \begin{bmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{bmatrix}$, $\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $\beta = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$, and I is 2×2 unit matrix .

(i) Show that $\sigma = \begin{bmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{bmatrix}$ commutes with H. [5]

(ii) Using the results of part (i) show that the one dimensional Dirac equation can be written as two coupled first order differential equations. [5]

7. Write the Dirac equation of an electron in the presence of a constant magnetic field $\vec{B} = B\hat{e}_z$. Determine its energy spectrum. [5]

8. The energy projection operators of the solutions of Dirac equation are defined as $\Lambda_\pm = \frac{m \pm \not{p}}{2m}$. Show that $\Lambda^2_\pm = \Lambda_\pm$, and $\Lambda_+\Lambda_- = 0$. How do these projectors act on the basic spinor solutions ? [3]

9. Starting with the covariant Dirac equation, derive the continuity equation and identify the probability and probability current densities. [3]