Indian Institute of Technology, Kharagpur
Department of Electronics and Electrical Communication Engineering
End-Spring Semester Examination–2011

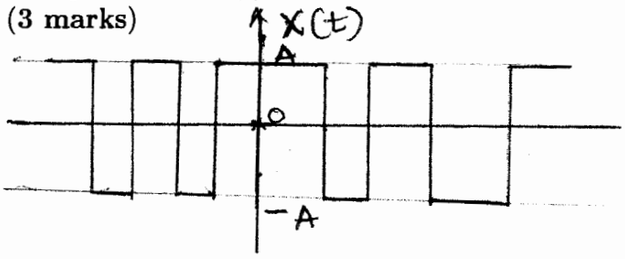
Program: B.Tech — Time: 3 hours
Subject: **EC 31002 Digital Communication** — Total Marks: 50

Answer all the questions. Total no.of pages: 3.

1. (a) Suppose that $S = \{\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_n}\}$ is an orthogonal basis for an inner product space and that $\mathbf{u}$ is any vector from the inner product space then, Prove that

$$\mathbf{u} = \frac{< \mathbf{u}, \mathbf{v_1} >}{\| \mathbf{v_1} \|^2}\mathbf{v_1} + \frac{< \mathbf{u}, \mathbf{v_2} >}{\| \mathbf{v_2} \|^2}\mathbf{v_2} + \cdots + \frac{< \mathbf{u}, \mathbf{v_n} >}{\| \mathbf{v_n} \|^2}\mathbf{v_n}. \quad \textbf{(2marks)}$$

(b) Consider a telegraph signal X(t) that assumes the values $\pm A$ with equal probability. A typical sample function of X(t) is shown in Fig. The average number of polarity switches (zero crossings) per unit time is α. The probability of having exactly k crossings in time τ is given by the poisson distribution. Find the autocorrelation and the power spectrum of X(t) and also sketch the autocorrelation and the power spectrum. **(3 marks)**



2. (i) Prove that

$$I(X_1, X_2, \ldots, X_n; Y) = \sum_{i=1}^{n} I(X_i; Y | X_{i-1}, X_{i-2} \ldots, X_1).$$

(1 mark)

(ii) A discrete memoryless source (DMS) X has five equally likely symbols. Construct a Huffmann code and find entropy H(X), average length of the code and code efficiency. (**3 marks)**

(iii) In the given codes, which code does not satisfy Kraft's inequality? **(1 mark)**

x_i	CODE-A	CODE-B	CODE-C	CODE-D
x_1	00	0	0	0
x_2	01	10	11	100
x_3	10	11	100	110
x_4	11	110	110	111

3. Consider a sinusoidal signal $m(t) = A\cos\omega_m t$, where $\omega_m = 2\pi f_m$ has been applied to a delta modulator with step size Δ.

(i) Under what condition the slope overload distortion will occur for this signal? (**1 mark**)

(ii) What is the maximum output signal-to-quantization-noise ratio under the assumption of no slope overload distortion?(**3 marks**)

(iii) Determine the output SNR for a 1 kHz sinusoid, sampled at 32 kHz, without slope overload, and followed by a 4 kHz post-reconstruction filter.(**1 mark**).

4. A Compact Disc (CD) recording system samples each of two stereo signals with a 16-bit ADC at 44.1 kilosamples/s.

   (a) Determine the output signal-to-noise ratio for a full-scale sinusoid. **(1 mark)**

   (b) If the recorded music is designed to have a crest factor (peak-to-rms ratio) of 20, determine the average output signal-to-quantizing noise ratio. **(1 mark)**

   (c) The bit stream of digitized data is augmented by the addition of error-correcting bits, substitution bits to aid the clock extraction by a phase-locked loop (PLL), and display and control bit fields. These additional bits represent 100% overhead; that is, 2 bits are stored for each bit generated by the ADC. determine the output bit rate of the CD recorder system. **(1 mark)**

   (d) The CD can record an hour's worth of music. Determine the number of bits recorded on a CD. **(1 mark)**

   (e) For a comparison, a good dictionary contain 1500 pages, 2 columns/page, 100 lines/column, 7 words/line, 6 letters/word and 6 bits/letter. Determine the number of bits required to describe the dictionary and estimate the number of comparable books that can be stored on a CD. **(1 mark)**

5. (a) What is need for a non-uniform quantizer? **(1 mark)**

   (b) Prove that the phase of the MSK signal is continuous. **(2 marks)**

   (c) What is the difference between FSK and MSK? **(1 mark)**

   (d) What is the unit of $\frac{E_b}{N_0}$? **(1 mark)**

6. One of three equally likely messages is communicated over a vector channel which adds a statistically independent zero-mean Gaussian random variable with variance $\frac{N_0}{2}$ to each transmitted vector component. Assume that the transmitter uses the signal vectors $s_k = (\cos\theta_k, \sin\theta_k)$, where θ_k takes values 0, $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ for the three messages.

   (a) Determine the minimum distance between any two points in the signal constellation. **(1 mark)**

   (b) Sketch the decision regions for the optimum receiver that minimizes the probability of symbol error. **(1 mark)**

   (c) Express the average probability of symbol error, P_e, in terms of the conditional error probabilities given the message index k, $P_{e|k}$. **(1.5 marks)**

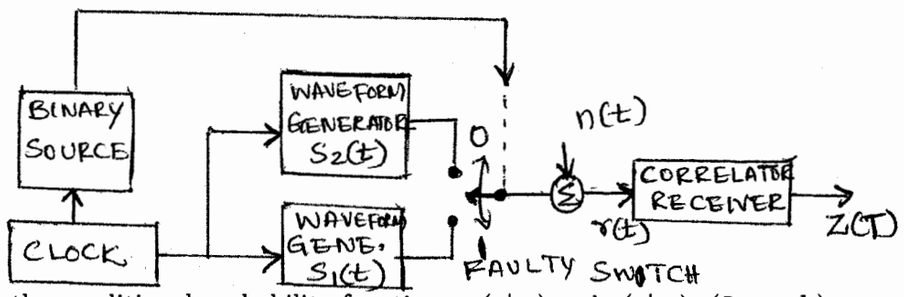
   (d) Show that the probability of symbol error is upper bounded by $2\,Q\left(\sqrt{\frac{3}{2N_0}}\right)$. **(1.5 marks)**

7. Consider an M-ary digital communication system where $M = 2^N$, and N is the dimension of the signal space. Suppose that the M signal vectors lie on the vertices of a hypercube that is centered at the origin. Determine average probability of symbol error. Assume that the noise is additive white Gaussian with power spectral density $\frac{N_0}{2}$ and the M-ary signal points are equiprobable. **(5 Marks)**

8. (a) Draw the early/late gate symbol synchronizer circuit with correct and early receiver timing diagrams. **(2+1 marks)**

(b) What is phase ambiguity problem in M-loop phase synchronizer? how do you avoid that? **(1+1 marks)**

9. A binary source with equally likely symbols controls the switch position in a transmitter operating over an AWGN channel as shown in Fig. This noise has two-sided spectral density $\frac{N_0}{2}$. Assume antipodal signals of time duration T seconds and energy E joules. The system clock produces a clock pulse every T seconds, and the binary source rate is $\frac{1}{T}$ bits/s. under *normal* operation, the switch is up when the source produces a binary zero, and it is down when the source produces a binary one. However, the switch is *faulty*. With probability p, it will be thrown in the wrong direction during a given T-second interval. The presence of a switch error during any interval is independent of the presence of a switch error at any other time. Assume that $\mathbf{E}[z(T)] = \pm E$, where **E** is the expectation operator.



(a) Sketch the conditional probability functions, $p(z|s_1)$ and $p(z|s_2)$. **(2 mark)**

(b) The correlator receiver observes r(t) in the interval (0,T). Sketch the block diagram of an optimum receiver for minimizing the bit error probability when it is known that the switch is faulty with probability p. **(2 marks)**

(c) Which one of the following two systems would you prefer to have?

$$p = 0.1 \; and \; \frac{E_b}{N_0} = \infty$$

or

$$p = 0 \; and \; \frac{E_b}{N_0} = 7 \; dB.$$

(1 mark)

10. Find the optimum filter frequency response $H_0(\omega)$ that maximizes the output SNR when the input noise is not a white noise. **(5 marks)**

********** BEST WISHES **********

Useful Formulae

- If $Y \sim \mathcal{N}(0,1)$, then

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}, \quad -\infty < y < \infty$$

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

- If X is Poisson random variable then the probability distribution function given by

$$Pr(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}, \quad k = 0, 1, 2, 3.., \; \lambda > 0$$

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