INDIAN INSTITUTE OF TECHNOLOGY

Date: FN/AN | Time: 3Hrs. Full Marks: 60 | No. of students: 72
Spring Semester 2008-2009 | Dept. Electronics & ECE | Sub. No. EC21004
2nd Year B.Tech.. | Sub. Name: Signals and Systems

Answer Q1. and any four from the rest. Answers should be brief, ***to the point and legible****. Sketches wherever appear should be neat and* ***properly labeled****. All parts of a question to be answered at one place.*

Q1. (a) Find if the following signal is periodic. If periodic, find its fundamental period.

$$x(t) = 4\sin(3t) + 3\sin(\sqrt{3}t)$$

(b) Plot the even and odd parts of the function, $x[n] = e^{-n/4}u[n]$.

(c) Find if the system with excitation $x(t)$ and response $y(t)$ and defined as follows is linear and invertible.

$$y(t) = x(t-5) - x(3-t)$$

(d) If $h[n]$ is impulse response of following system, find $h[50]$ and $h[100]$.

$$8y[n] + 6y[n-1] = x[n]$$

(e) Plot the convolution product, $y[n] = x[n]*h[n]$, where $x[n] = u[n] - u[n-4]$ and $h[n] = \delta[n] - \delta[n-2]$

(f) Find if the value of a is fixed where probability distribution function of random variable X is defined as

$f_X(x) = ae^{-0.2x}$ for $x \geq 0$ and zero, elsewhere 6 x 2 = 12

Q2. (a) Why ROC description is a must in definition of z-Transform?

(b) State and prove the final-value theorem in z-Transform.

(c) Comment on the frequency selectivity from pole-zero diagram for $H(z) = z/(z-a)$ for three different cases of $a = 0.9, 0.1, -0.9$ respectively.

(d) Solve using z-transform the difference equation $y[n+2] - (3/2)y[n+1] + (1/2)y[n] = (1/4)^n$, for $n \geq 0$ where $y[1]=4$, $y[0]=10$. 2 + 2 + 3 + 5

Q3. (a) Compare Laplace Transform, Fourier Transform, z-Transform.

(b) Solve using Laplace Transform : $x''(t) + 7x'(t) + 12x(t) = 0$ for $t > 0$ where, $x(0-) = 2$, $x'(0-) = -4$

(c) The Laplace Transform of a continuous-time system is given as $H(s) = (s^2 + 40s + 300)^{-1}$. Consider an equivalent sampled-data system $H(z)$ that uses sampling frequency f_s and zero-order-hold Digital-to-Analog Converter. Find $H(z)$ and compare step responses of two systems for (i) $f_s = 10$ Hz and (i) $f_s = 100$ Hz. Plot step responses in each case and calculate error, if any. 3 + 3 + 6

Q4. (a) Compare DTFT with DFT.

(b) Using Parseval's theorem, find the signal energy of $x(n) = \text{sinc}(n/10)\sin(2\pi n/4)$

(c) Plot and compare magnitude and phase of

(i) DTFT of $0.25\text{sinc}(n/16)$ and (ii) DTFS of $0.25\text{sinc}(n/16) * \delta_{32}[n]$ where, '*' stands for convolution.

(d) Calculate 4-point DFT $X[k]$ for sequence $\{x[n]\}=\{1, -1, 0, 1\}$. Hence, compute IDFT of $\{X[k]\}$. 2 + 2 + 4 + 4

Q5. (a) Find closed form expression for the output $y[n]$ if input $x[n] = u[n]$ and impulse response $h[n] = n(7/8)^n u[n]$.

(b) Draw Direct Form II block diagram of the system described as

$$3y[n] + 2y[n-2] = 4x[n] - 2x[n-2] + x[n-3]$$

(c) A system is described by $ny[n] - 8y[n-1] = x[n]$. Comment on its linearity, time-invariance, BIBO stability and invertibility. 4 + 3 + 5

Q6. (a) Define random variable and random process.

(b) The probability of error in transmission of digital data through a noisy channel is 0.001. If each transmission is an independent event, what is the probability that more than two data are erroneous out of 100 transmissions?

(c) Consider the random process, $X(t) = 0.5\cos(0.2t + \theta)$ where θ is a uniform random variable in the range $[-\pi, \pi]$. Find if $X(t)$ is WSS.

(d) In what sense, Weiner-Hopf filter is optimal? The message, $X(t)$ a random process with signal power $S_M(\omega) = (1 + \omega^2)^{-1}$ passes through a channel that corrupts the message with white noise of power $S_M(\omega) = 0.1$. Find the optimal filter that will maximize the output signal-to-noise-ratio. 2 + 3 + 3 + 4

------- X -------