End Semester Examination
Dept. of Electronics and Electrical Communication Engineering, IIT Kharagpur

Subject: Analog Communication
Subject Code: EC31001

Duration: 3 Hours
Total Marks: 100

There are 5 questions each carrying 20 marks. All the questions are to be attempted.

1. (a) Suppose that X is a random variable described by an exponential pdf

$$f_X(\alpha) = \lambda e^{-\lambda\alpha};\ \alpha \geq 0.$$

Define a function q which maps real numbers into integers by q(x) = the largest integer less than or equal to x. (This function is denoted by $q(x) = \lfloor x \rfloor$) The function q is a form of quantizer, it rounds all real numbers downward to the nearest integer below the input real number. Define the following two random variables:

the quantizer output

$$Y = q(X)$$

and the quantizer error

$$\epsilon = X - q(X).$$

Note: By construction ϵ can only take on values in [0; 1).
(i) Find the probability mass function $p_Y(k)$ for Y .
(ii) Find the expectations E(X) and E(Y).
(iii) Derive the probability density function for ϵ.

(b) Consider the coding scheme of Figure 1 in which the encoder is a simple quantizer inside a feedback loop. H and G are causal linear filters. Show that if the filters are chosen so that

$$\hat{X}_n = \tilde{X}_n + q(\epsilon_n),$$

Then

$$|X_n - \hat{X}_n| = |\epsilon_n - q(\epsilon_n)|$$

And hence

$$E(|X_n - \hat{X}_n|^2) = E(|\epsilon_n - q(\epsilon_n)|^2),$$

that is, the overall error is the same as the quantizer error. Suppose that G is fixed. What must H be in order to satisfy the given condition? What constraint is there on G so that H will be causal? (For the purpose of this problem, the "quantizer" can be any nonlinear memory-less mapping.)

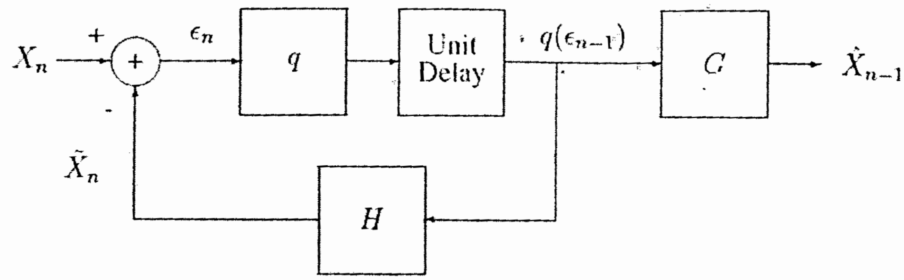


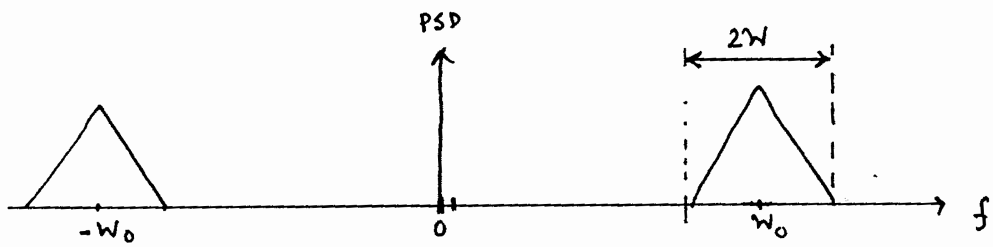
Figure 1: Coding scheme

10+10

2. (a) Using the sampling theory, prove that a band limited signal f(t) (of bandwidth W) can be represented as follows:

$$f(t) = \sum_{n=-\infty}^{\infty} f(n/2W)\frac{\sin(2Wt-n)}{\pi(2Wt-n)}$$

(b) suppose a modulated signal is represented as follows:



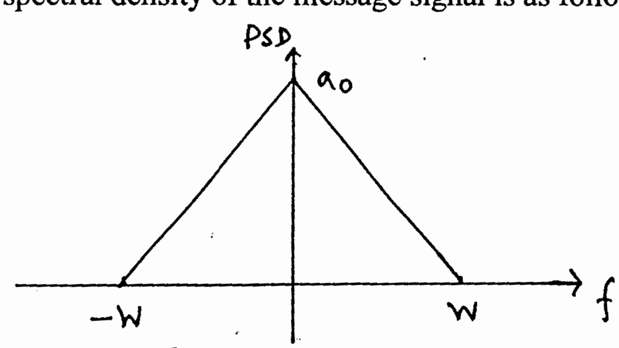
Prove that a band pass sampling of frequency (f_s) = $\dfrac{W_0}{\left\lfloor W_0/2W \right\rfloor}$ followed by a low pass filter of bandwidth W can reconstruct the original modulating signal.

(c) If 'f_s' in the above example is shifted by 'd', then what are the allowed values of 'd', so that the signal can still be reconstructed from its samples without distortion.

7+6+6

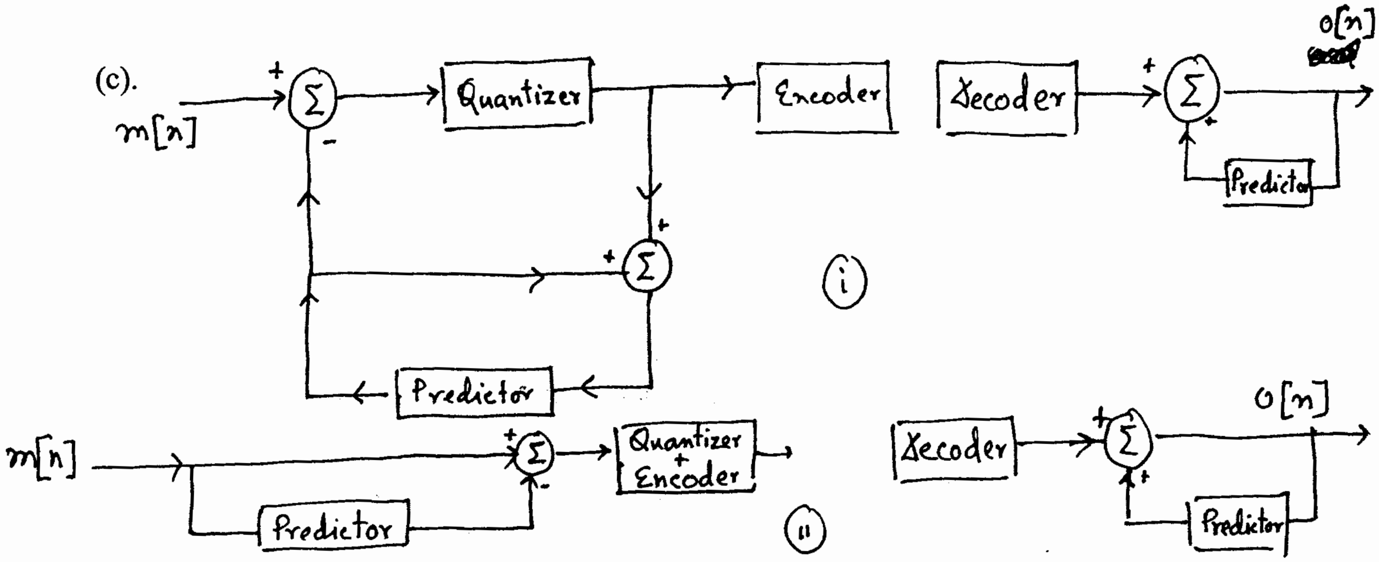
3. (a) For a message signal m(t) =Acos(wt) derive the condition to avoid slope overload for a delta modulation, if the step size is E and the sampling frequency is f_s.

(b) If the power spectral density of the message signal is as follows:



Derive the expression for the minimum step size to avoid slop overloading for a delta modulation given the sampling frequency f_s.

(c).



For the 2 different DPCM modulator, show that the demodulator output of the first one will always be the quantized version of m[n];whereas, the second one might have completely different output signal not necessarily equal to either the signal itself or its quantized sample.

5+10+5

4(a) Let m(t) be the message signal to be transmitted using single sideband modulation. The power spectral density of m(t) is

$$S_M(f) = a\frac{|f|}{W} \quad \text{for } |f| \le W$$
$$0 \quad \text{Otherwise}$$

Where, a and W are constant. If white Gaussian noise of zero mean and power spectral density $N_0/2$ is added to the SSB modulated wave at the receiver input, then derive the expression for output signal to noise ratio. Also derive expression for SSB figure of merit.

(b) Suppose that the transfer function of the pre-emphasis and de-emphasis filters of an FM system are scaled as follows:

$$H_{pe}(f) = k(1+\frac{jf}{f_0})$$

$$H_{pe}(f) = (1/k)(\frac{1}{1+\frac{jf}{f_0}})$$

The scaling factor k is to be chosen so that the average power of the emphasized message signal is same as the original message signal m(t).

(i) Find the value of k that satisfies the requirement for the case when the power spectral density of the message signal m(t) is

$$S_M(f) = \frac{S_0}{1+(f/f_0)^2} \quad \text{for } |f| \le W$$
$$0 \quad \text{Otherwise}$$

(ii) What is the corresponding value of the improvement factor I provided by the filter; where $I = \dfrac{2W^3}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 \, df}$

12+8

5(a) For a stationary random process x(t):
Prove that the Fourier transform of $R_x(\tau)$ is $S_x(f)$.

Where, $R_x(\tau) = \overline{x(t)x(t+\tau)}$ and $S_x(f) = \lim_{T \to \infty} \dfrac{\overline{|X_T(f)|^2}}{T}$

Here, $\bar{x}$ denotes the statistical averaging of a random variable x and $X_T(f)$ is the Fourier transform of the truncated signal $x_T(t)$.

$x_T(t) = x(t)$ for $|t| \le T/2$

0 Otherwise

(b) For a signal with power spectral density $S_m(f)$ in the presence of channel noise of power spectral density $S_n(f)$; derive the expression for the receiver input filter transfer function, so that the overall effect of channel noise and signal distortion due to filtering get minimized.

10+10