INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Department of Electrical Engineering

Mid-Sem Semester, September 25, 2012

Time: 2 Hours | EE 41013: Digital Signal Processing | Full Marks: 120

Answer all questions. Make suitable assumptions wherever necessary

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Q.1 A sequence has the discrete-time Fourier transform $X(e^{j\omega}) = \dfrac{1-a^2}{(1-ae^{-j\omega})(1-ae^{j\omega})} \quad |a|<1$ **[15]**

(a) Find the sequence $x[n]$. (b) Calculate $\dfrac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)\cos(\omega)\,d\omega$.

Q.2 An ideal low-pass filter with zero delay has impulse response $h_{lp}[n]$ and frequency response

$$H_{lp}(e^{j\omega}) = \begin{cases} 1, & |\omega| < 0.2\pi, \\ 0, & 0.2\pi \le |\omega| \le \pi \end{cases} \qquad \textbf{[15]}$$

A new filter is defined by the equation $h[n] = \dfrac{\sin(0.1\pi\, n)}{\pi\, n} h_{lp}[n]$. Determine the equation for the frequency response $H(e^{j\omega})$, and approximately plot the nature for $|\omega| < \pi$.

Q.3 The autocorrelation sequence of a signal $x[n]$ is defined as $R_x[n] = \sum_{k=-\infty}^{\infty} x^*[k]x[n+k]$ **[5+10]**

(a) Show that for an appropriate choice of the signal $g[n]$, $R_x[n] = x[n] * g[n]$, and identify the proper choice for $g[n]$.

(b) Show that the Fourier transform of $R_x[n]$ is equal to $|X(e^{j\omega})|^2$.

Q.4 In Fig.Q4

(a) Prove that the overall system in the dotted box is LTI.

(b) Determine an expression for $H(e^{j\omega})$, the frequency response of the overall system in terms of $H_1(e^{j\omega})$, the frequency response of the internal LTI system.

(c) Find and approximately show the nature of $H(e^{j\omega})$ for the case when the frequency response of the internal LTI system is $H_1(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c, \\ 0, & \omega_c < |\omega| \le \pi. \end{cases}$ **[15]**

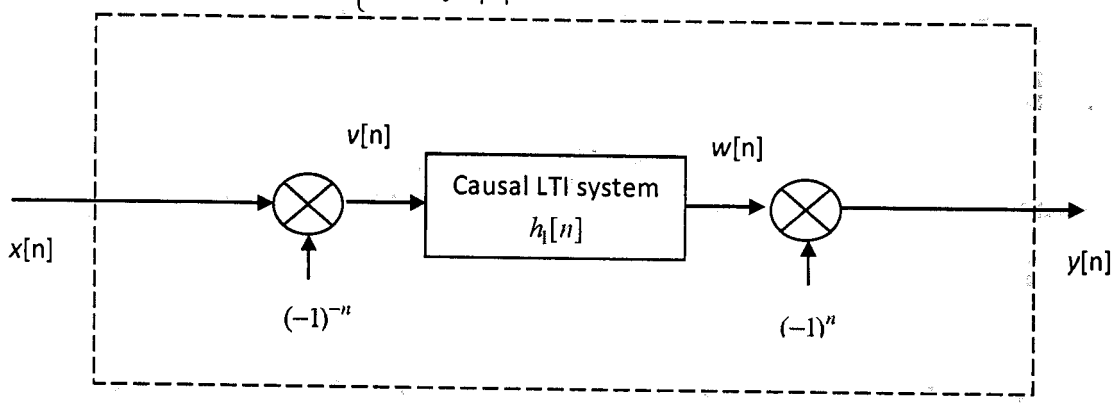

Fig.Q.4

P.T.O.

Q.5 Determine the Z-transform of each of the following sequences including the ROC **[8+7]**

$$(a)\sum_{k=-\infty}^{\infty}\delta[n-4k] \qquad (b)\frac{1}{2}[e^{j\pi n}+\cos(\frac{\pi}{2}n)+\sin(\frac{\pi}{2}+2\pi n)]u[n]$$

Q.6 Consider an LTI system that is stable and for which $H(z)$, the z- transform of the impulse response, is given by $H(z)=\dfrac{3-7z^{-1}+5z^{-2}}{1-\frac{5}{2}z^{-1}+z^{-2}}$. Find the output when the input is a unit step sequence. **[15]**

Q.7 Consider the system shown in the Fig.Q.7. For each of the following input signals $x[n]$, indicate whether the output $x_r[n]=x[n]$. **[15]**

$$(a)\,x[n]=\cos(\pi n/4) \qquad (b)\,x[n]=\left[\frac{\sin(\pi n/8)}{\pi n}\right]^2$$

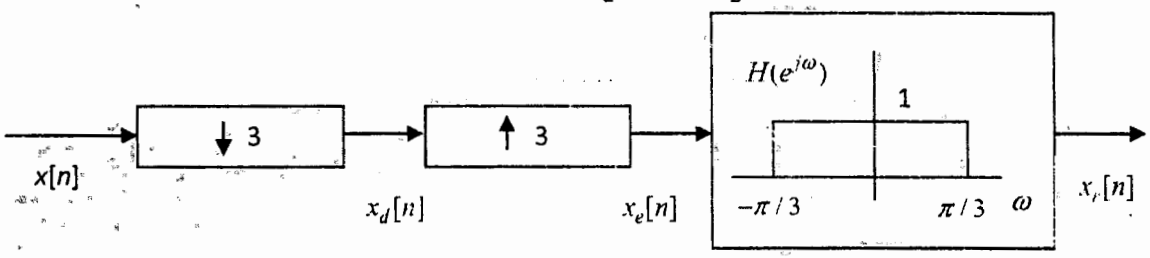

Fig.Q.7

Q.8 In the system of the Fig.Q.8, assume that $X_c(j\Omega)=0$ for $|\Omega|>\pi/T$. Determine and approximately plot the magnitude and phase of the frequency response of the discrete-time LTI system such that the output is $y_r(t)$ is the running integral of the input, i.e. $y_r(t)=\int_{-\infty}^{t}x_c(\tau)d\tau$. **[15]**

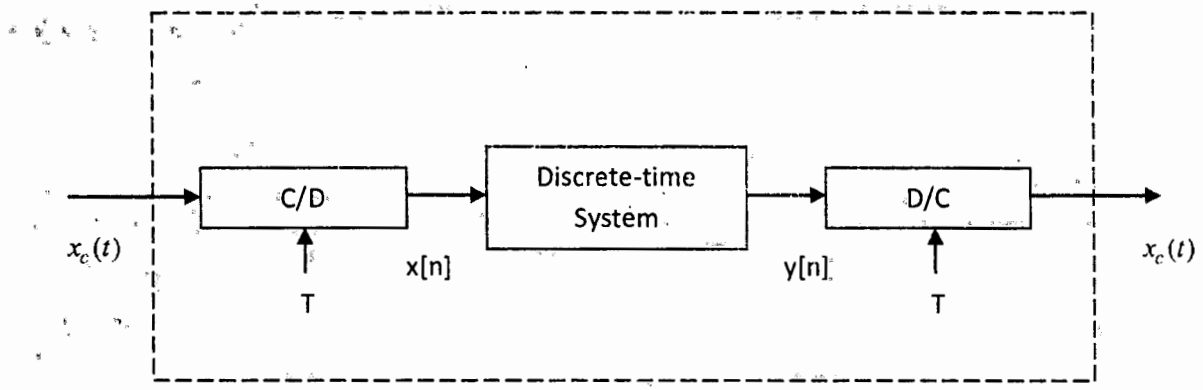

Fig.Q.8