1) The closing values of the price of ACC Ltd. is given in the table.

Date	Stock price
23-Mar-18	1527.8
22-Mar-18	1546.4
21-Mar-18	1554.35
20-Mar-18	1556
19-Mar-18	1559.7
16-Mar-18	1565.55

From the most satisfactory estimate of the kth lag autocorrelation,

a) find $var(\bar{z})$
b) construct the ACF plot.
c) Will this estimate of the first lag autocorrelation be the same as the first PACF? Why?
d) Assume that there are 24 trading days in a month. If the daily closing price of ACC Ltd. for two more months is added to the existing sample given in the above table, assuming that the autocorrelations remain unchanged as calculated from the existing sample whenever $j \leq 4$, and $\rho_j = 0$ for $j > 4$, obtain the approximate standard error for r_j, $j > 4$.
(22 marks)

2) For any stationary process, the autocorrelation matrix is positive-definite. Prove or disprove. (4 marks)

3) Obtain the maximum likelihood estimates of an AR(1) process. (12 marks)

4) a) The following is the output of Box-Ljung test. Provide an analysis and inference of the output.

To lag	Q statistic	p-value
5	4.091378	0.2517645
10	7.833827	0.4498687
15	11.985102	0.5288659
20	19.736039	0.3478749

25	28.147803	0.210244
30	33.460065	0.2192169

b) Suppose more than one ARIMA(p,d,q) model fits a data. Which model will you select? Why?

(10 marks)

5) Obtain the autocovariance function for the following process:

$(1-0.6B-0.3B^2)(1-0.5 B^s)a_t$ $s > 4$ (13 marks)

6) If $\widehat{y_0}$ represents the point estimate of a future value y_0 in a simple linear regression of y on x, obtain an expression for $E(y_0 - \widehat{y_0})^2$ (11 marks)

7a) Is the following process stationary?

b) Is invertible?

$$\tilde{Z}_t - 0.4\,\tilde{Z}_{t-1} = a_t - 0.25a_{t-1} + 0.8a_{t-2}$$

Obtain the

    i)   First four ψ weights
    ii)   First four π weights
    iii)   Autocovariance and Autocorrelation function
    iv) 2nd PACF

(13 marks)

8) Let $\{a_t\}$ is an iid N(0,1) noise. Define

$$Z_t = \begin{cases} a_t; & \text{if t is even} \\ \dfrac{(a_{t-1}^2 - 1)}{\sqrt{2}}; & \text{if t is odd} \end{cases}$$

Show that $\{Z_t\}$ is uncorrelated with mean 0 and variance 1, but not iid noise with mean 0 and variance 1. (5 marks)