Economics of Growth (HS 30087)

Department of Humanities and Social Sciences

Final Examination (Autumn 2018)

Answer all questions

1. **Consider a standard OLG model with utility function, $u(c_t, c_{t+1}) = log(c_t) + \beta log(c_{t+1}), 0 < \beta < 1$. Here, $c = \frac{C}{L}$ represents percapita consumption. The production function is, $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, 0 < \alpha < 1$. Assume there is no population growth, capital depreciates fully and gross growth rate of technology is, $\frac{A_{t+1}}{A_t} = A$. Define, $x_t = \frac{X_t}{L_t}$. (25)**

a) Find out optimal, k_{t+1} and explain your result intuitively. (9)

b) Define, $G_k = \frac{k_{t+1}}{k_t}$ and $G_y = \frac{y_{t+1}}{y_t}$. Show that, $G_k = G_y = A^{\frac{1}{1-\alpha}}$. Can you show this even without solving the model explicitly? Explain your result intuitively. (3+3+2)

c) Show that, $k_t = \left[\frac{\beta A_t(1-\alpha)}{(1+\beta)A^{\frac{1}{1-\alpha}}}\right]$. (3)

d) What happens when $\beta = 1$ and $r = 0$? Explain intuitively. (5)

2. **Consider a simple Ramsey model. Assume that, the rate of growth of population is $n \in (0,1)$, and there is no technology growth. Suppose, production function is, $y(t) = f(k(t))$, strictly concave. Utility function is, $U = \int_0^{\infty} e^{-(\rho-n)t} u(c(t)) dt$. Utility function is strictly concave. Depreciation rate of capital is, $\delta \in (0,1)$. Market is perfectly competitive. (25)**

a) Derive the resource constraint in percapita form. (2)

b) State the problem of social planner and set up the Current Value Hamiltonian. (4+2)

c) Derive the First order Necessary Conditions. (3)

d) Show that, steady state capital stock is strictly less than golden rule level of capital stock. Argue that this is required for the stability of the model. (3+2)

e) Mathematically show that the system is Saddle Path stable. Identify the state, co-state and the jump variable(s) of the model. (6+3)