INDIAN INSTITUTE OF TECHNOLOGY

Date: April , 2012 | Time: 3Hrs. Full Marks: 100 | No. of students: 120
Spring Semester 2011-2012 | Dept. Electronics & ECE | Sub. No. EC21004
2nd Year B.Tech.. | Sub. Name: Signals and Systems

← PLS. READ INSTRUCTIONS CAREFULLY →

*Answer **all** five questions. Answers should be **brief, to the point and legible**. Sketches wherever appear should be **neat** and **properly labeled**. Justification, steps, assumptions must be **unambiguously** mentioned. All parts of a question must be answered **at one place.***

Q1. (a) If input to a system, defined by $h[n] = [1, 2, 4]$ is $x[n] = [1, 1, 0, -2]$ then find output of the system. Is this system stable?

(b) Solve the difference equation

$$y[n+2] - \frac{3}{2}y[n+1] + \frac{1}{2}y[n] = (1/4)^n \quad , \text{ for } n \geq 0 \text{ where } y[0] = 10 \text{ and } y[1] = 4$$

(c) Given, a system represented by unit sample response $h_1(n)$ is a Low Pass Filter. Show if another system with unit sample response $h_2(n) = (-1)^n h_1(n)$ represents a Low Pass Filter/ High Pass Filter/Band Pass Filter/Band Stop Filter.

(d) What is the origin of aliasing in time domain? How can this be prevented?

(e) For a linear system with impulse response $h(t)$, consider the input $x(t)$ as wide sense stationary process with mean μ_X and autocorrelation $R_X(\tau)$. Find mean and autocorrelation of the output $y(t)$.

5+5+3+3+4

Q2. (a) Realize $y[n] - 0.5\,y[n-2] = x[n] + 0.25\,x[n-1] - 0.5\,x[n-2]$ in hardware using only two delay elements.

(b) What is transposition theorem? Using transposition theorem give an alternate representation of 2(a). Comment on the difference between realization 2(a) and 2(b).

(c) For both realizations of 2(a) and 2(b) give $x[n] = [1, -2, 3]$ as input and show how output is generated in each case by giving values at different nodes at different time instances.

(d) A 90 degree phase shifter is a system with frequency response

$$H(e^{j\omega}) = -j \text{ for } 0<\omega<\pi \text{ and } H(e^{j\omega}) = j \text{ for } -\pi<\omega<0$$

Find unit sample response of the system.

5+5+4+6

Q3. (a) Define z-transform and state the significance of Region of Convergence. State and prove the accumulation property of z-transform.

(b) Consider a single pole stable system. Take three different values of the pole and show how the system responds to unit step sequence input in each of these three cases and plot them.

(c) For three cases in 3(b) find frequency response of the system and comment on findings of 3(b) and 3(c), if they are related in anyway.

(d) The output $y[n]$ of a discrete-time LSI system is found to be $2(1/3)^n u[n]$ when the input $x[n] = u[n]$.

Find (i) the impulse response $h[n]$ of the system and
(ii) the output $y[n]$ when input $x[n]$ is $(1/2)^n u[n]$.

5+4+5+6

Q4. (a) What is circular convolution? How linear convolution can be obtained from circular convolution?

(b) Compute DFT coefficients $X[k]$ for $x[n] = [1, 1, 0, -1]$. Hence, compute inverse DFT of $X[k]$.

(c) Consider, $x[n]$ of 4(b) and $y[n] = [1, 0, -1, 1]$. Find $z[n]$ which is circular convolution of $x[n]$ $y[n]$. Using DFT, show how $X[k]$, $Y[k]$ and $Z[k]$ are related.

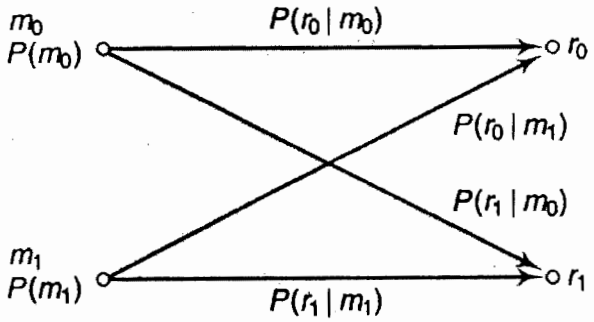
(d) Find N-point DFT of the sequence $x[n] = \cos(n\omega_0)$ where $0 \leq n \leq N-1$. Compare DFT coefficients $X[k]$ when $\omega_0 = 2\pi m/N$ where m integer and when $\omega_0 \neq 2\pi m/N$. Explain the difference.

3+5+6+6

Q5. (a) What is noise? What is the significance of Signal to Noise Ratio (SNR) in the context of Signals & Systems?

(b) Give mathematical definition of 'orthogonality'. Show how addition of noise affect the distinguishability of signal. From this, comment how design of a system is affected due to addition of noise or what are the associated trade-offs.

(c) For the communication channel shown in following figure, it is given that $P(r_0|m_0) = 0.9$, $P(r_1|m_0) = 0.1$, $P(r_0|m_1) = 0.4$, and $P(r_1|m_1) = 0.6$. Find the probability of error for an optimal receiver designed for above system. State the rule adopted for the decision at the receiver end.



(d) If noise affecting the channel for the system defined in 5(c) is gaussian in nature, show using histogram plot how conditional probability is calculated for above and decision threshold is obtained for optimum receiver.

(e) Define Aurocorrelation and Power Spectral Density. How are they related?

3+5+5+4+3

→ END OF Q. PAPER ←