CE 21005: Solid Mechanics
Mid Term Examination, Autumn 2012
IIT, Kharagpur

Answer All Questions

1. A rigid cubic solid is immersed in a fluid with weight density ρ. Recall that a stationary fluid exerts a compressive pressure ρH at a depth of H.

(a) Write down the expressions for the traction vector acting on each face of the cube. (5 marks)

(b) Calculate the resultant force due to the tractions acting on the cube and show that the vertical force is equal to the weight of the fluid displaced by the cube. (5 marks)

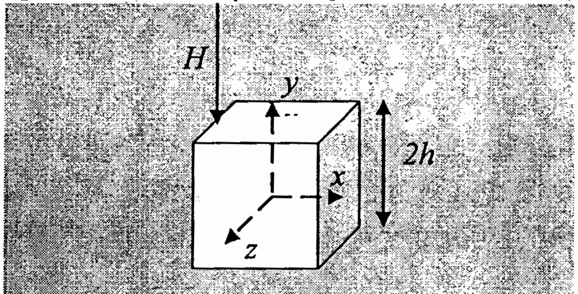

Fig. 1

(c) Write down the stress matrix at the point $(h,0,0)$ on the cube in the (x,y,z) system with origin at. the cube center. Evaluate the principal stresses and maximum shear stress at the point. (5 marks)

(d) Finally consider a counter clockwise rotation of 45 degrees about the z axis of (x,y,z) system. Find the stress components at the above point in the new system. (5 marks)

2.(a) Show that the principal directions of the deviatoric stress matrix are identical to that of the full stress matrix and the principal values differ by the hydrostatic pressure.(4 marks)

(b) What are the compatibility conditions and why are they necessary? (4 marks)

(c)What is material stability? When will an elastic material be stable? Show that for a stable linear elastic isotropic material, the Poisson's ratio v must lie between -1 and 0.5. (5 marks)

(d)What is the St. Venant's principle and why is it important? (3 marks)

(e)Torsional loads applied to a solid cylinder about its axis do not lead to warping deformations. Explain why with any relevant equations. (4 marks)

3. (a) Show that a stress function of the form $\varphi(x, y)$ does not strictly satisfy the plane stress compatibility conditions. (5 marks)

(b) Next consider a plane strain case represented by the dam model in Fig. 2. The bottom of the dam is fixed to the rigid floor. Write down the traction boundary conditions in terms of the stress components on faces OA and OB of the dam. Consider the Airy's stress function represented by: $\phi = \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 y + \frac{C_3}{2}xy^2 + \frac{C_4}{6}y^3$

Show that this is a valid stress function. Use the boundary conditions on OA and OB to find the values of the constants C_1 through C_6 and hence show that:

$$\sigma_{xx} = -\rho y, \sigma_{yy} = -\frac{2\rho x}{\tan^3 \beta} + \frac{\rho y}{\tan^2 \beta}, \sigma_{xy} = -\frac{\rho}{\tan^2 \beta} x \quad \text{(15 Marks)}$$

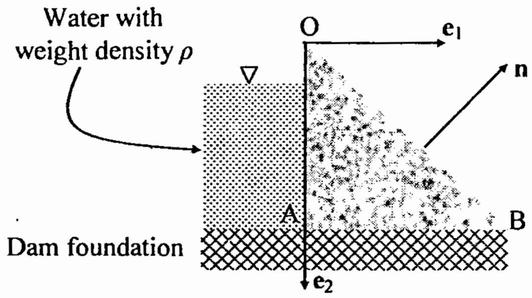

Fig. 2

4.(a) In a thick walled cylinder made of a linear elastic isotropic material and subject to symmetrical loading what are the non-zero displacement components and why? At sections far removed from the ends what are the non-zero strain components? If a strain component is zero, explain why it is so. Write the equations of equilibrium and compatibility equations for such a cylinder in cylindrical coordinates (10 marks)

(b) A thick solid cylinder open at the ends, is subject to internal and external pressure of 100 MPa and 40 MPa respectively. The cylinder has an inside diameter of 20 mm and an outside diameter of 40 mm. What are the radial, circumferential and axial stresses at the inner radius of the cylinder? Given that the cylinder is made of aluminium with Young's modulus E = 72 GPa and Poisson's ratio = 0.33, determine the axial, radial and circumferential strains at the inner radius. (10 marks)

5. A shaft 60 mm in diameter is supported at bearing O and carries a 750 mm diameter pulley with a weight of 2.5 kN at the overhanging end and loads as shown in Fig. 3. At the bearing, calculate the axial and shear stress in an infinitesimal element at location A in the cross section of the shaft. Draw the Mohr's circle for the state of stress at this point and from the Mohr's circle find the two principal stresses and the maximum shear stress. Again, using the Mohr's circle, find the angle the normal to the plane on which the maximum principal stress acts makes with the axis of the shaft. (20 Marks)

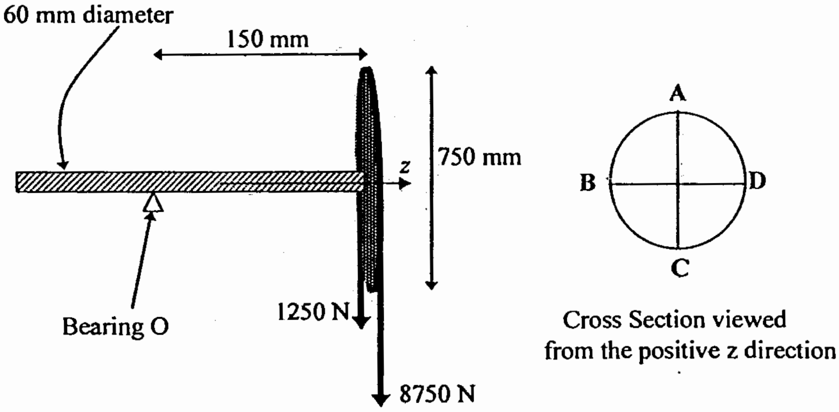

Fig. 3