Department of Aerospace Engineering, IIT Kharagpur
End-Autumn Semester Examination, 2015-16

2^{nd}-Year B.Tech and Dual Degree

Time: 3 h | **AE21003 Dynamics for Aerospace Engineers** | **Marks: 50**

Attempt ALL Questions. Answer all parts of a question CONSECUTIVELY & IN SEQUENCE
Marks: Each part of each question carries two marks

1a) What are the quantities conserved in any central force motion? How many independent coordinates are necessary to describe the path of a particle undergoing central force motion? Why?

b) In a Rutherford type experiment an alpha particle approaches, and is repelled by, a gold nucleus. Assuming the gold nucleus to be fixed, what can you say about the trajectory of the alpha particle?

c) List all possible factors which, in your opinion, cause the tyre on the rear wheel of a bicycle to wear out faster than the tyre on the front wheel. Try to explain how each factor contributes to the wearing out.

2a) A pendulum consists of a small but heavy bob of mass m suspended from an *elastic* cord of nominal length l. The stiffness of the cord is k N/m. If this pendulum is set in oscillatory motion in a plane, how many degrees of freedom will the bob have, assuming it does not spin?

Obtain, in plane polar coordinates, the equations of motion of the pendulum bob assuming the cord is never slack.

b) A compound pendulum consists of a slender, rigid, rod of length l and mass m, to the end of which is welded a uniform sphere of radius R and mass M. If this pendulum oscillates in one vertical plane then determine its time period for small oscillations. Note that R is *not* small compared to l and the moment of inertia of a uniform sphere about an axis passing through its centre is $\frac{2}{5}MR^2$.

3a) Starting from the equation of its trajectory in plane polar coordinates, show that the eccentricity ε of the path of a space vehicle moving in the gravitational field of a large spherical body is given by: $\varepsilon = \dfrac{v_1^2 r_1}{GM} - 1$, where G is the universal gravitational constant, M is the mass of the attracting body, r_1 is the *closest* distance of the vehicle from the centre of attraction and v_1 is the corresponding velocity.

b) If the trajectory of approach of a certain space vehicle towards an astronomical body has eccentricity $\varepsilon = 1.3$, by what factor should its velocity be reduced so that it goes into a circular orbit of radius r_1 about the body? At what point along the trajectory should this velocity reduction occur?

4a) What constraint must be imposed upon a system of particles to make it behave like a rigid body?

b) Write down the mathematical expression for the angular momentum of a system of particles relative to a point A.

c) What constraint must be imposed on the motion of a point A relative to a rigid body B such that it is possible to write the components of the relative angular momentum of B about A as:

$$\vec{\mathbf{H}}_A = \overline{\overline{\mathbf{I}}}_A \cdot \vec{\omega}$$

where the symbols have their usual meanings?

d) Among the set of points A which satisfy the criterion of Q.4c), for what subset of points can you write for a rigid body:

$$\vec{\mathbf{M}}_A = \left(\frac{d\vec{\mathbf{H}}_A}{dt}\right)_{XYZ}$$

where $\vec{\mathbf{M}}_A$ is the moment of all external forces about point A, $\vec{\mathbf{H}}_A$ is the relative angular momentum of the system of particles about A and XYZ is an inertial frame of reference?

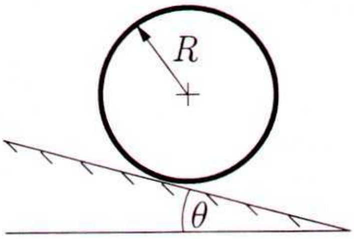
e) If $Axyz$ is a frame of reference whose origin is always at A, what choice of angular motion of $Axyz$ allows us to write the equation in Q.4d) as:

$$\vec{\mathbf{M}}_A = \overline{\overline{\mathbf{I}}}_A \cdot \frac{d\vec{\omega}}{dt} + \vec{\Omega} \times \left(\overline{\overline{\mathbf{I}}}_A \cdot \vec{\omega}\right)$$

where $\vec{\omega}$ is the angular velocity of the body and $\vec{\Omega}$ the angular velocity of the $Axyz$ axes, both relative to the inertial axes XYZ? Why?

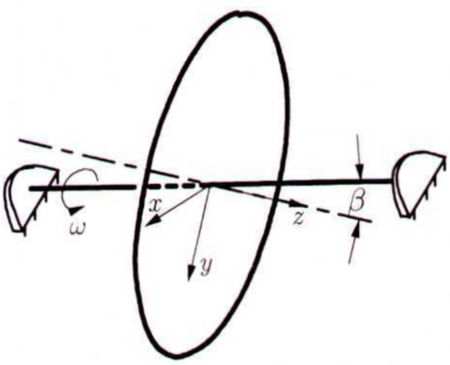
5) A uniform solid sphere of mass $M = 2.0$ kg and radius $R = 0.08$ m rolls down a $\theta = 36.9°$ slope (see figure overleaf). The coefficient of sliding friction between the sphere and the slope is 0.15.

a) Draw a free body diagram of the rolling sphere showing i) the forces acting on it and ii) the moving axis system in which you wish to write the equations of motion. Why is it not necessary to use body fixed axes in this problem?



b) What are the i) linear and angular accelerations of the sphere, and ii) the magnitude and direction of the friction force? (Given $I_C = \frac{2}{5} MR^2$.)

6) A disk of mass M and radius R is mounted on a horizontal shaft which passes through its centre but makes a small angle β with its axis.



a) What is the moment acting on the disk about its mass centre when the shaft turns with a *constant* angular speed ω?

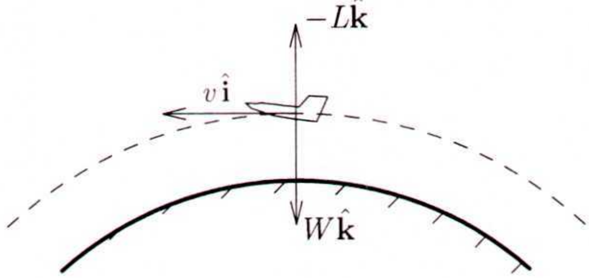
b) What does this moment do? How is the moment transmitted to the disk?

c) If the shaft is mounted between bearings spaced a distance L apart, what are the reactions at these bearings due to the rotational motion of the disk-shaft assembly?

d) How do these reactions vary with the time?

e) What is the notable feature about the magnitude of the moment? From this feature, what can you say about the effect of even small misalignments on high speed rotating machinery, such as gas turbine rotors?

7) A futuristic hypersonic aircraft flies at a speed $\vec{\mathbf{v}} = 15,000\,\hat{\mathbf{i}}$ km/hr at an altitude of 30 km above the ground (the Earth's surface) and follows an arc of a great circle. Velocity vector $\vec{\mathbf{v}}$ is always parallel to the ground. The $\hat{\mathbf{i}}$-$\hat{\mathbf{j}}$-$\hat{\mathbf{k}}$ system shown is rigidly fixed at the mass centre of the aircraft, with the $\hat{\mathbf{i}}$ axis parallel to the ground, the $\hat{\mathbf{j}}$ axis pointing into the page and the $\hat{\mathbf{k}}$ axis pointing vertically downward. W is the weight of the aircraft and L is the aerodynamic force normal to $\vec{\mathbf{v}}$, called lift. It opposes the weight and makes it possible for heavier-that-air craft to fly.



a) The *load factor* is defined as the ratio L/W. i) Show that the load factor is unity when the aircraft flies parallel to the surface of a *flat Earth*. ii) Now calculate the load factor accounting for the curvature of the Earth, as shown in the figure. iii) What would be the percentage error in the load factor if the Earth was assumed to be flat? Ignore the Earth's rotation. ($R_{\text{Earth}} = 6,370$ km).

b) If we wish to keep the error, arising out of ignoring the curvature of the Earth, in the load factor to less than 5% then what is the maximum speed of an aircraft for which we may assume the Earth to be flat?

c) Determine the angular velocity of the aircraft and the acceleration of its mass centre.

8a) What is *torque free motion*? Where are you likely to find bodies in torque free motion? What property does the angular momentum vector about the mass centre of a body in such motion have?

b) Explain with appropriate sketches and physical arguments why, when a child's top is spinning rapidly, it does not fall down to the floor.

c) Why does giving an axial spin to rifle bullets and unguided missiles improve their range?

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