AEROSPACE ENGINEERING DEPARTMENT

INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Mid Autumn Semester Examination (2010-2011).
Time: 2 Hrs

17.09.2010 AN **AE21001 Introduction to Aerodynamics**

2^{nd} Year B. Tech. (H) **No of Students: 38**

Answer any four full questions. All questions carry equal marks.

Assumptions, if required, can be made with appropriate justifications.

Some important vector identities that may be required:

$$\nabla\times(s\vec{u})=s\nabla\times\vec{u}-\vec{u}\,\nabla s$$

$$\nabla\times(\vec{u}\times\vec{v})=\vec{v}\quad\nabla\vec{u}+\vec{u}\nabla\cdot\vec{v}-\vec{u}\nabla\vec{v}-\vec{v}\nabla\cdot\vec{u}$$

$$\nabla^2\vec{u}=\nabla\cdot\nabla\vec{u}=\nabla(\nabla\cdot\vec{u})-\nabla\times(\nabla\times\vec{u})$$

$$\vec{u}\nabla\vec{u}=\frac{1}{2}\nabla(\vec{u}\cdot\vec{u})+(\nabla\times\vec{u})\times\vec{u}$$

$$(\vec{u}\cdot\nabla)\vec{u}=\frac{1}{2}\nabla(\vec{u}\cdot\vec{u})-\vec{u}\times\nabla\times\vec{u}$$

Notations have their usual meaning unless specified otherwise.

1(a) Show that the stress tensor in a fluid at rest is everywhere isotropic and only normal stresses act.

(b) A closed vessel full of water is rotating with constant angular velocity Ω about a horizontal axis. Show that the surfaces of equal pressure are circular cylinders whose common axis is at a height g/Ω^2 above the axis of rotation.

2(a) Considering the Eulerian velocity at two neighbouring positions in space show that the fluid velocity is the superposition of a uniform velocity, a pure straining motion without change in volume, an isotropic expansion and a rigid body rotation.

(b) Derive the mathematical statement that satisfies the requirement of conservation of mass of a moving fluid. Use this equation to show that a flow can be treated as incompressible if density changes due to change in pressure are negligibly small. What restrictions the equation imposes on the velocity field under this condition?

3(a) Show that the local rate of change of vorticity in a flow is given by the sum total of vorticity convected with the flow, vorticity diffused by the viscous action and local redistribution of vorticity due to rotation and stretching. Assume incompressible flow with constant properties.

(b) Show that a vortex filament cannot end within a fluid.

4(a) Define singly-connected and multiply-connected domains. How many irreconcilable circuits can be drawn in doubly-connected region? If the velocity field in a doubly connected region of space is solenoidal and irrotational, what conditions are required to be satisfied so that the velocity field can be determined uniquely?

(b) The velocity components for a 2D incompressible flow are given by $u = e^x \cosh y$ and $v = -e^x \sinh y$. Find the streamlines.

5(a) Under what conditions a flow can be approximated as incompressible and inviscid? Show that in such a flow the rate of change of circulation around a material closed curve is zero if the body force is field is conservative.

(b) What is a Newtonian fluid? Write down the complete stress tensor for a Newtonian fluid. The isotropic part of the stress tensor is called mechanical pressure – why? Is this quantity different from equilibrium or thermodynamic pressure?

6(a) The Lagrangean position coordinates of a fluid particle in a 2D motion are given by $x = x_0 e^{kt}, y = y_0 e^{-kt} + x_0\left(1 - e^{-kt}\right)$. Find the trajectory of the particle that was occupying the position (x_0, y_0) at t_0. What are the Eulerian velocity components for this flow? Is the flow steady? Can this be a kinematically possible incompressible flow?

(b) The velocity components

$$u_r(r,\theta) = -U_\infty\left(1 - \frac{R^2}{r^2}\right)\cos\theta$$

$$u_\theta(r,\theta) = U_\infty\left(1 + \frac{R^2}{r^2}\right)\sin\theta$$

satisfy the equation of motion for a 2D inviscid, incompressible flow. Find the pressure associated with this velocity field. The parameters U_∞ and R are constants.