INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Mid-Autumn Semester 2018-19

Date of Examination: 25.09.2018 **Session: AN** **Duration: 2 hrs** **Full marks: 30**
Subject No.: PH20003 **Subject: PHYSICS – II** **Department: Physics**

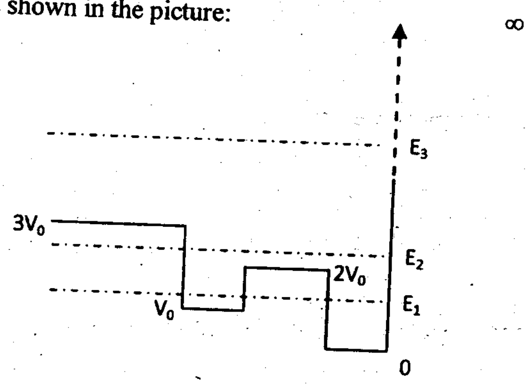
Instructions: *Attempt all questions. Corresponding marks are indicated.*

1. Show that the eigenvcetors of $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ are orthogonal. *2*
2. Find the eigenvectors of $i\frac{d}{dx}$ that are periodic in interval $0 \le x \le 2\pi$. Find also the corresponding eigenvalues and show that the eigenvectors form a complete orthonormal set. [Hint: Choose any two arbitrarily different eigenfunctions to prove the orthonormality.] *2+1+1*
3. Check whether the operator defined as 'multiplication by e^{ix}' is Hermitian. *1*
4. The energy of a linear harmonic oscillator is

$$E_x = \frac{p_x^2}{2m} + \frac{cx^2}{2}.$$

Assuming that the uncertainty product is at its maximum and that $\Delta x \sim x$ and $\Delta p_x \sim p_x$, show that the minimum energy of the oscillator is $\frac{1}{2}h\nu$, where $\nu = \frac{1}{2\pi}\sqrt{\frac{c}{m}}$. *2*

5. Consider the 1-D normalized wavefunctions $\psi_0(x)$ and $\psi_1(x)$ such that $\psi_0(-x) = \psi_0(x) = \psi_0^*(x)$ and $\psi_1(x) = N\frac{d\psi_0}{dx}$, where N is a constant. Calculate $\langle\psi_0|\psi_1\rangle$ and the expectation value of momentum in the state $\psi_0(x)$. *½ + ½*
6. State and describe the postulates of quantum mechanics. *2*
7. The general wavefunction ψ of a certain operator is given as a linear combination $\psi = \sum_n c_n\psi_n$ of the complete orthonormal set ψ_n of the eigenvectors. In an experiment, the probability P_n of finding the particle in state ψ_n is given by $P_n = \langle\psi|\psi_n\rangle$. Show that $P_n = |c_n|^2$. Also show that if the ψ_ns are eigenfunctions of the Hamiltonian, then ψ is also an eigenfunction. *1+2*
8. Giving qualitative arguments, copy the picture below and draw the schematic wavefunctions for the three total energies E_1, E_2, and E_3 corresponding to the potential as shown in the picture:



Assume $E_3 >> 3V_0$. *1 + 1 + 1 + 1*

9. Consider the $n = 1$ and $n = 3$ states of a particle in a box (infinite potential well). Find $\langle\psi_1|\psi_3\rangle$. [Hint: Use the relation cos A + cos B = [cos (A+B) + cos (A-B)]/2.] *2*
10. Verify that the eigenfunction and eigenvalue for $n = 1$ state of a quantum harmonic oscillator follow the Schrödinger equation. *3*
11. Calculate the location at which the radial probability density, given as $P_{nl}(r)\,dr = R^*_{nl} R_{nl} \cdot 4\pi r^2\,dr$, is maximum for the $n = 2$, $l = 1$ state of the Hydrogen atom. Then calculate the expectation value $\bar{r}$ of the radial coordinate in this state. *4*
12. All four of the functions $e^{im_l\phi}$, $e^{-im_l\phi}$, $\cos m_l\phi$ and $\sin m_l\phi$ are solutions to the equation for $\Phi(\phi)$. Find which out of these are also the eigenfunctions of $\hat{L}_z$. *2*