Indian Institute of Technology Kharagpur

Date: 24/11/2011 (AN) Time: 3 hours Full Marks: 50 No of Students: 31

End-Autumn Semester: 2011-12 Department: HSS Subject No: HS20003

Five-Year Integrated M.Sc. in Economics Subject Name: Statistics for Economics

**Instruction: Answer all the questions. Statistical Tables will be supplied on request.**

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1. Fill in the blanks with appropriate word(s): $^1/_2 \times 6 = 3$

   (a) If the coefficient of skewness negative, mode is _____ than median.
   (b) Mathematical expectation of a constant is _____.
   (c) The variance of a standardized normal variable is _____.
   (d) If an estimator approaches to the true parameter with increase in the sample size, it is _____.
   (e) A linear combination of two independent $\chi^2$ variables follows _____ distribution.
   (f) The square of a t-distributed random variable with k degrees of freedom has _____ distribution.

2. Answer the following questions: $2 \times 8 = 16$

   (a) State how moments are used to describe the characteristics of a variable.
   (b) Prove that the 4th central moment, $\mu_4 = \mu_4 - 4\mu_3\mu_1 + 6\mu_2(\mu_1)^2 + (\mu_1)^4$ where $\mu_i$ is the i$^{th}$ raw moment.
   (c) The mean, median and coefficient of variation of weekly wages of a group of workers are Rs. 45, Rs. 42, and Rs. 40 respectively. Find the mode and the coefficient of skewness for the distribution of wages.
   (d) If a coin is tossed three times and $X$ is a random variable denoting the number of heads occurring, determine the mathematical expectation and the variance of $X$.
   (e) If $X$ and $Y$ are two independent random variables with standard deviations $\sigma_X$ and $\sigma_Y$ respectively, show that the correlation coefficient between $X$ and $X+Y$ is $\frac{\sigma_X}{\sqrt{\sigma_X^2 + \sigma_Y^2}}$.
   (f) Prove that if $X$ and $Y$ are two independent random variables, $E(XY) = E(X).E(Y)$.
   (g) State the important properties of a normal distribution.
   (h) If the life of electric bulb (in hours) follows normal distribution with mean 155 and standard deviation 19, what is the probability that the life of a bulb will be between 117 and 193 hours?

3. (a) Compute the appropriate measure of skewness for the following data:

| Age (Years) | < 20 | 20-25 | 25-30 | 30-35 | 35-40 | 40-45 | 45-55 | > 55 |
|---|---|---|---|---|---|---|---|---|
| No. of Employees | 13 | 29 | 46 | 60 | 112 | 94 | 45 | 21 |

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   (b) Consider the following probability density function of the random variable $X$:

$f(x) = ax$ for $0 < x \le 1$

$= a$ for $1 < x \le 2$

$= -ax + 3a$ for $2 < x \le 3$

$= 0$, otherwise

Find $P(X \le 1.5)$ 3

(c) Consider the following probability density function for the random variable $X$:

$$f(x) = \frac{x}{2} \text{ if } 0 < x \leq 1$$
$$= \frac{1}{2} \text{ if } 1 < x \leq 2$$
$$= \frac{(3-x)}{2} \text{ if } 2 < x \leq 3$$
$$= 0 \text{ otherwise}$$

Derive the corresponding cumulative distribution function. 3

(d) Prove that for $n$ independent random variables, $X_1, X_2, \ldots\ldots\ldots, X_n$,

$$Var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} Var(X_i)$$ 3

(e) Consider two independent random variables $X$ and $Y$. If $X$ has a set of discrete values 0, 1, 2, 3, and 4 with probability $\frac{1}{5}$ each, and $Y$ has a set of discrete values 0, 1, 2, and 3 with probability $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{3}$, and $\frac{1}{6}$ respectively, find $P(XY \leq 3)$. 3

4 (a) What are the possible errors of complete census approach? How is stratified random sampling different from multi-stage random sampling? 4

a) The table below gives yields of four varieties of a crop in 3 blocks of a district:

| Block | Variety | | | |
|---|---|---|---|---|
| | I | II | III | IV |
| A | 9 | 10 | 9 | 10 |
| B | 12 | 11 | 9 | 11 |
| C | 11 | 12 | 10 | 12 |

Examine if there is significant difference in yield between the varieties or between the blocks. If so, find out which pairs of variety differ significantly. 6

b) Assume that agricultural production per hectre (PROD) depends on irrigation facilities (IRRI), and use of high yielding variety seeds (HYV), fertilizer (FERT) and pesticides (PEST) per hectre. The table below presents results of the regression model, $YIELD_t = \alpha + \beta_1 IRRI_t + \beta_2 HYV_t + \beta_3 FERT_t + \beta_4 PEST_t + u_t$ estimated for the period 1971-72 to 2001-02:

| Variable | Coefficients | Standard Error |
|---|---|---|
| Intercept | 26.7191 | 36.2780 |
| IRRI | 173.2291 | 182.0039 |
| HYV | 2.3170 | 72.2182 |
| FERT | 30.7852 | 33.4469 |
| PEST | 62.3593 | 23.9610 |

Further, explained sum of squares (ESS) and residual sum of squares (RSS) are 10971.88 and 1247.71 respectively. Carry out necessary statistical tests for acceptability of the regression results and interpret the results. 6