Subject: PH41014 Condensed Matter Physics-I

Mid-Semester Examination: Spring 2015-16

Marks=30. Time 2 hrs

Alert: Please answer in the same sequence as in the question paper

1. The primitive translation vectors of a lattice are given below

$\boldsymbol{a} = \sqrt{3}\, a/2\, \hat{\boldsymbol{x}} + a/2\, \hat{\boldsymbol{y}}$; $\boldsymbol{b} = -\sqrt{3}\, a/2\, \hat{\boldsymbol{x}} + a/2\, \hat{\boldsymbol{y}}$; $\boldsymbol{c} = c\, \hat{\boldsymbol{z}}$, where $\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}$ are unit vectors along the x, y, z directions and a, c are real constants.

(a) Show that the volume of the primitive cell is $\sqrt{3}\, a^2 c/2$ (4)

(b) Find the primitive translations of the reciprocal lattice. (5)

(c) Draw the first and 2nd Brilluoin zones of the lattice. (6)

2. A linear chain of masses M are coupled to the next one on either side by springs of constant K. Consider a longitudinal wave $u_n = u \cos(\omega t - kna)$ on this one-dimensional lattice, where n is the site number and a is the lattice constant.

(a) Find the dispersion relation for ω. *(3)*

(b) Show that the total energy carried by the wave is
$E = 1/2\, M \Sigma_n \left(\partial u_n/\partial t\right)^2 + 1/2\, K \Sigma_n \left(u_n - u_{n+1}\right)^2$ *(5)*

(c) Using $u_n = u \cos(\omega t - kna)$, show that the time-averaged (over a period) energy $\langle E \rangle$ per atom is

$1/4\, M \omega^2 u^2 + 1/2\, K u^2 (1 - \cos(ka))$.

Use the dispersion relation in this and show that the average potential and kinetic energies are the same. (7)