Indian Institute of Technology Kharagpur QUESTION-CUM-ANSWERSCRIPT	Stamp/Signature of the Invigilator

END-SEMESTER EXAMINATION											SEMSETER (Autumn-2015)			
Roll Number											Section		Name	
Subject Number	M	A	1	0	0	0	1				Subject Name		Mathematics I	
Department/Centre/School														

Important Instructions and Guidelines for Students

1. You must occupy your seat as per the Examination Schedule/Sitting Plan.
2. Do not keep mobile phones or any similar electronic gadgets with you even in the switched off mode.
3. Loose papers, class notes, books or any such materials must not be in your possession; even if they are irrelevant to the subject you are taking examination.
4. Data book, codes, graph papers, relevant standard tables/charts or any other materials are allowed only when instructed by the paper-setter.
5. Use of instrument box, pencil box and non-programmable calculator is allowed during the examination. However, the exchange of these items or any other papers (including question papers) is not permitted.
6. Write on both sides of the answer-script and do not tear off any page. Use last page(s) of the answer-script for rough work. Report to the invigilator if the answer-script has torn or distorted page(s).
7. It is your responsibility to ensure that you have signed the Attendance Sheet. Keep your Admit Card/Identity Card on the desk for checking by the invigilator.
8. You may leave the Examination Hall for wash room or for drinking water for a very short period. Record your absence from the Examination Hall in the register provided. Smoking and consumption of any kind of beverages is strictly prohibited inside the Examination Hall.
9. Do not leave the Examination Hall without submitting your answer-script to the invigilator. **In any case, you are not allowed to take away the answer-script with you**. After the completion of the examination, do not leave your seat until invigilators collect all the answer-scripts.
10. During the examination, either inside or outside the Examination Hall, gathering information from any kind of sources or exchanging information with others or any such attempt will be treated as '**unfair means**'. Don't adopt unfair means and also don't indulge in unseemly behavior.
11. Please see overleaf for more instructions

Violation of any of the above instructions may lead to severe punishment.

Signature of the Student

To be Filled by the Examiner

Question Number	1	2	3	4	5	6	7	Total
Marks Obtained								

Marks Obtained (in words)	Signature of the Examiner	Signature of the Scrutineer

Instructions and Guidelines to the Students appearing in the Examination

1. The question-cum-answer booklet has 36 pages and 7 questions.
2. All questions are compulsory.
3. Answer each question in the space provided below to that question only. Otherwise it will not be checked.
4. No additional answer sheet will be provided.
5. Use the space for rough work given in the booklet only.
6. After the completion of the examination do not leave the examination hall until the invigilator collects the booklet.

1(a). Find the positive real value of λ, for which the co-efficient of x^{64} in the Maclaurin expansion of $(x+\lambda^2)e^x$ is $\frac{5}{\lfloor 63}$. [2M]

1(b). Find the intervals for which the curve $y=e^x(\sin x+\cos x),\ x\in(0,2\pi)$ is convex upwards (concave downwards) or convex downwards (concave upwards).

[3M]

1(c). Find the value(s) of α for which the radius of curvature of the curve $x=\alpha(t+\sin t)$, $y=\alpha(1-\cos t)$ at $t=2\pi$ is 1. [2M]

2(a). Find $f_{xy}(0,0)$ and $f_{yx}(0,0)$ for the function

$$f(x,y) = \begin{cases} \dfrac{xy(x^2+2y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

where $f_{xy} \equiv \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$ and $f_{yx} \equiv \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$. [4M]

2(b). A container of capacity 64 m^3 in the form of a cuboid with an open top is to be made of thin sheet of metal. Calculate, with justification, the dimensions of the container if it has to use minimum possible amount of metal sheet.

[4M]

3(a). Find the value of λ such that

$$(2xe^y + 3y^2)\frac{dy}{dx} + (3x^2 + \lambda e^y) = 0$$

is exact. Using this value of λ, find the general solution of the differential equation.

[3M]

3(b). Solve the differential equation

$$y + \frac{d}{dx}(xy) = x(\sin x + \ln x)$$

[2M]

3(c). Solve the differential equation

$$\frac{dy}{dx} + \frac{y}{x}\ln y = \frac{y}{x^2}(\ln y)^2$$

[2M]

4(a). Using the method of variation of parameters, find the general solution of the following non-homogeneous differential equation

$$\frac{d^2y}{dx^2} + 4y = 3 \operatorname{cosec} x$$

[4M]

4(b) Find the solution of the following homogeneous differential equation

$$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0$$

with the conditions: $y(0) = 0,\ y'(0) = -1,\ y\left(\frac{\pi}{2}\right) = -1,\ y'\left(\frac{\pi}{2}\right) = -1.$ [3M]

5(a). Find the general solution of the differential equation

$$(3+2x)^2\frac{d^2y}{dx^2}-2(3+2x)\frac{dy}{dx}+4y=8x$$

[3M]

5(b). Find the general solution of the following system of differential equations

$$\frac{dy_1}{dx}=y_1+4\,y_2$$
$$\frac{dy_2}{dx}=2y_1-y_2$$

(WITHOUT reducing it to a differential equation of 2^{nd} order) [4M]

6(a). Discuss the continuity of the function $f(z) = \mathrm{Arg}(z)$ (Principal Argument) at a point $z = x$, where x is real and negative. [1M]

6(b). If $u(x, y) = x^2 - y^2$ and $v(x, y) = 3x^2y - y^3$ are Harmonic functions in a domain D, then show that the function $F(x, y) = (u_y - v_x) + i(u_x + v_y)$ is analytic in D. [3M]

6(c). Let f(z) be defined as

$$f(z) = \begin{cases} \dfrac{\mathrm{Re}(z^3) + i\,y^3}{|z^2|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

Test the C-R equations at the origin. Is this function differentiable at the origin? Justufy.

[3M]

(7)(a). Use the M-L inequality to show that

$$\left|\int_{\Gamma} \frac{dz}{z^2+1}\right| \leq \frac{1}{2\sqrt{5}}$$

where Γ is the line segment joining the points 2 and 2+i. Show the details of your work.

[2M]

7(b). Evaluate the integral

$$\int_{\Gamma} \frac{e^{\pi i z}\ dz}{2z^2-5z+2}$$

where Γ is the unit circle $|z| = 1$ that orients counter-clockwise. [3M]

7(c). Show that the integral

$$\int_{\Gamma} \frac{z\ dz}{z+1}$$

is path independent and hence evaluate it. Here Γ is any curve lying in the upper half plane {z: Im (z)>0} joining the points -1+2i and 1+2i. [2M]