INDIAN iNSTITUTE OF TECHNOLOGY, KHARAGPUR

End-Autumn Semester 2018-19

Date of Examination: 22-11-2018 Session: AN Duration: 3 hr

PH31011/PH41011 Subject: Quantum Mechanics I

Department of Physics

The paper has **7 questions, answer all.**

1. The ket space of a system is spanned by the three orthonormal eigenkets $|a_1>$,$|a_2>$ and $|a_3>$ of a Hermitian operator $\hat{A}$. Consider the kets

$$|\alpha >= (|a_1> + i|a_2> + |a_3>)/\sqrt{3}$$

$$|\beta >= (1/\sqrt{4})|a_1> + (1/\sqrt{4})|a_2> + (1/\sqrt{2})|a_3> .$$

(a.) Calculate $<\alpha|\alpha>$, $<\alpha|\beta>$ and $<\beta|\beta>$.

(b.) Using the $|a_i>$ basis, calculate the matrix representations of $|\alpha>$, $<\alpha|$,$|\beta>$, $<\beta|$,$\hat{X} = |\alpha><\beta|$ and $\hat{X}^\dagger$.

(c.) Calculate $Tr(\hat{X}\hat{X}^\dagger)$.

2. Given a SHO with mass m and angular frequency ω, using energy eigenkets $|n>$ as basis vectors

(a.) Calculate the matrix representation of $\hat{x}$, $\hat{p}$, $\hat{x}^2$ and $\hat{p}^2$.

(b.) Use these to calculate the uncertainities $<(\Delta x)^2>$ and $<(\Delta p)^2>$ for $|1>$.

Considering the state $|\alpha, t>$ with $|\alpha, 0>= (|0> + i|2>)/\sqrt{2}$

(c.) Calculate $|\alpha, t>$

(d.) Calculate $<\hat{x}>(t)$ and $<\hat{p}>(t)$ corresponding to the state $|\alpha, t>$.

3. Consider the correlation function defined by

$$C(t) = <\hat{x}(t)\hat{x}(0)>$$

where $\hat{x}(t)$ is the position operator in the Heisenberg picture. Evaluate the correlation function explicitly for the ground state of the one dimensional simple harmonic oscillator with angular frequency ω.

4. Calculate the propagator $K(x,t;x^{'},t^{'})$ $(t^{'} < t)$ for a particle of mass m inside an infinite potential well with $V(x) = 0$ for $0 \leq x \leq a$ and infinite outside. (You can leave the answer as an infinite sum.)

5. Consider an ensemble of spin half particles where 50% have spin $(1/2)\hbar$ along $+z$ and 50% have spin $(1/2)\hbar$ along $+x$.

   (a.) Calculate $\hat{\rho}$ the density operator for this ensemble.

   (b.) Calculate $Tr(\rho)$.

   (c.) Calculate the ensemble averages $[\hat{S}_z]$ and $[\hat{S}_y]$.

6. The spin of an electron is in the state $|\alpha >= (|1/2, 1/2 > + |1/2, -1/2 >)/\sqrt{2}$.

   (a.) What are the expectation values of $\hat{J}^2$, $\hat{J}_x$, $\hat{J}_y$, $\hat{J}_z$?

   (b.) The electron is rotated through an angle θ around the z axis. Find the new state of the electron.

   (c.) How do the expectation values calculated earlier change under this rotation?

7. Let $|1, -1 >$, $|1, 0 >$ and $|1, 1 >$ be three eigenkets of the operator $\hat{J}^2$ and $\hat{J}_z$, all corresponding to $l = 1$ and with the values $m = -1, 0, 1$ respectively.
   (a.) Calculate the matrix representation of the operators $\hat{J}_+$ and $\hat{J}_-$.
   (b.) Calculate the matrix representation of the operators $\hat{J}_x$ and $\hat{J}_y$.

Planck Constant: $h = 6.62607004 \times 10^{-34}\,\mathrm{m^2 kg}\, s^{-1}$