Indian Institute of Technology, Kharagpur
End-Autumn Semester Examination: 2018–2019

Date of Examination: *April,2018* *Session FN/AN ,* *Duration: 3 Hrs,*
Subject. No. MA60056 *No. of Registered Students 63*
Subject Name: REGRESSION AND TIME SERIES MODEL
Department: Mathematics *TOTAL MARKS: 50*
Specific Chart, graph paper log book etc. required.... NO...
*Special Instruction:***(1)Answer all parts of a question in consecutive places. (2)Full credit will be given to the answers correct up to two decimal places.**

ANSWER ALL THE QUESTIONS

1. Let $\{Y_t\}$ be a stationary time series with finite auto-covariance function. Define $X_t = \sum_{j=-\infty}^{\infty} \psi_j Y_{t-j}$, where $\sum_{j=-\infty}^{\infty} |\psi_j| < \infty$. Find the auto-covariance between $\{X_t\}$ and $\{X_{t+h}\}$, if it exists for $h \in \mathbb{N}$. [6]

2. Consider $AR(1)$ process $X_t = \phi X_{t-1} + Z_t$ where $Z_t \sim WN(0, \sigma^2)$ and $|\phi| < 1$. Prove that the distribution of X_t is a limiting distribution of an $MA(\infty)$ process. [6]

3. Let $\{Y_t\}$ be a stationary time series with finite variance σ^2 and $\sum_{h=-\infty}^{\infty} |\gamma(h)| < \infty$. Find the value of $Var(\frac{1}{n}\sum_{t=1}^{n} Y_t)$ when $n \uparrow \infty$. [6]

4. Consider an $AR(1)$ process $X_t = 0.35X_{t-1} + Z_t$ where $Z_t \sim WN(0, 2.5)$. Suppose the observed values of $X_4 = 2.4$, $X_6 = 1.3$, but X_5 is missing.
(a)Approximate the missing value of X_5 based on the available information which minimizes the least squared error.
(b) Find the mean squared error in approximation of X_5. [3+3]

5. Consider a time series $\{X_t\}$ with zero mean and $\gamma(0) = 1, \gamma(1) = 0.72, \gamma(2) = -0.67, \gamma(3) = 0.53, \gamma(4) = -0.45, \gamma(5) = 0.36$. When X_5 is predicted by (X_4, X_3, X_2, X_1) then the coefficients are respectively $(0.78, 0.42, 0.35, 0.28)$. Now the observed value of $(X_4, X_3, X_2, X_1, X_0)$ is to be used to predict the value of X_5.
(a) Find the coefficient of X_0 by Durbin-Levinson Algorithm.
(b) Find the ratio of the mean squared error when X_5 is predicted by $(X_4, X_3, X_2, X_1, X_0)$ to the same when the prediction is done by (X_4, X_3, X_2, X_1). [4+2]

6. (a) What is the definition of positive definite matrix?
(b) What is the definition of positive semidefinite function?
(c) Show that an auto-covariance function of a time series is a positive semidefinite function.
. [2+2+2]

7. For the model $\underset{\sim}{\mathbf{Y}} = \mathbf{X}\underset{\sim}{\beta} + \underset{\sim}{\epsilon}$, where $\underset{\sim}{\epsilon} \sim N(\underset{\sim}{\mathbf{0}}, \sigma^2\mathbf{I}_n)$, $\underset{\sim}{\mathbf{Y}} \in \mathbb{R}^n$, $\underset{\sim}{\beta} \in \mathbb{R}^{(k+1)}$use Jackknife method to test at 5% level for the null hypothesis that the observation y_5 is not an outlier based on the following estimates. Residual $e_5 = 2.10$, $MSResudual = 1.04$ and 5^{th} diagonal element of projection matrix $h_{55} = 0.036$, where $n = 25$, $k = 6$. State the conclusion. [6]
. [P.T.O]

8. State TRUE or FALSE in complete words for the following statements. [8]

(a) If $\{X_t\}$ is a weakly stationary time series, then X_5 and X_7 are identically distributed.

(b) If $\{W_t\}$ is white noise , then W_i and W_j are always independently distributed for $i \neq j$.

(c) When $X_t - 0.5X_{t-1} = Z_t + 2Z_{t-1}$ and $\{Z_t\}$ is WN then $\{X_t\}$ is an invertible time series

(d) Innovations algorithm for prediction uses the prediction error of the past data.

(e) $(X_t - X_{t-2}) = (1 - B)(1 + B)X_t$

(f) Diagonal elements of $(\mathbf{X}\mathbf{X}^T)^{-1}$ are indicators of leverage.

(g) $R^2_{adjusted}$ is always a strictly monotone function of the number of regressors.

(h) For the model $\underset{\sim}{\mathbf{Y}} = \mathbf{X}\underset{\sim}{\beta} + \underset{\sim}{\epsilon}$, $|\mathbf{X}\mathbf{X}^T| = 0$ always implies multicollinearity.

***************** THE END *****************