

Mathematical Methods II (PH41008)
Spring Mid-Semester Examination 2015-2016

No. of students: 67

M. Sc./Department of Physics

M.M: 36]

[Duration: 2 hrs.

Answer all questions.

Q. 1: Identify groups among the following. Write only yes or no. Cite proper reasons in case of 'no'. [3]

- a) The set of *non*-negative integers under addition.
- b) The set of integer powers of 2 under multiplication.
- c) The set of all complex numbers under multiplication.

Q. 2: Choose the correct option for the following. [3]

- a.) Number of distinct *non*-abelian groups of order 31 is
i) 0 ii) 1 iii) 2 iv) 3.
- b.) Number of elements in the symmetry group of a square is
i) 2 ii) 6 iii) 8 iv) 12.
- c.) Tetrahedral group (only with pure rotations) has 4 conjugate classes. The dimensions of the irreducible representations of this group are
i) 1, 1, 1 & 3 ii) 1, 2, 2 & 2 iii) 1, 2, 3 & 3 iv) 1, 2, 4 & 4.

Q. 3: A finite group has order 42. What are the possible orders of the elements of its proper subgroups? [2]

Q. 4: If $a = (146)(325)$ and $b = (153642)$, find bab^{-1} and aba^{-1} . [2]

Q. 5: $(12)(34)$ and $(13)(24)$ are two elements of an abelian group. Do these elements belong to the same conjugate class? Explain your answer. [2]

Q. 6: Find the number of conjugate classes for the symmetric group S_5 by writing at least one element of each class. [2]

Q. 7: Separate the elements of alternating group A_4 in conjugate classes. Does it have any subgroup of order 4. If the answer is yes, then identify that subgroup and separate A_4 in terms of cosets of this subgroup. Is this subgroup invariant? [8]

Q. 8: Find out possibilities of existence of simple groups of orders 168 and 280 by Sylow analysis. [4]

Q. 9: Is regular representation of S_3 reducible? If yes, decompose it into irreducible representations of S_3 . [5]

Q. 10: Find the dimension and any two characters of the representation of S_5 corresponding to the following Young diagram. [5]

