INDIAN INSTITUTE OF TECHNOLOGY

Date : Time : 2 Hrs. Full Marks 60 No. of Students 100

Mid-Spring Sem. Dept. of E & ECE Sub. No. EC21006

Sub.: Electromagnetic Engineering

Instructions : 1) ATTEMPT ALL THE QUESTIONS
2) MAKE NECESSARY ASSUMPTIONS WITH JUSTIFICATIONS, IF NECESSARY
3) ATTEMPT ALL THE PARTS OF A QUESTIION AT ONE PLACE

1. A light source inside a translucent sphere of 20 cm diameter causes a light flux density at the spherical surface 1,000 $cos^2(\theta/2)\hat{a}_r$ lumens/m^2. **5**

   a) In what direction is the flux density a maximum?

   b) Determine the angle $\theta = \theta_0$ at which the flux density is one-half its maximum value.

   c) Determine the angle $\theta = \theta_1$ such that one-half the total light flux is emitted within the case $\theta < \theta_1$.

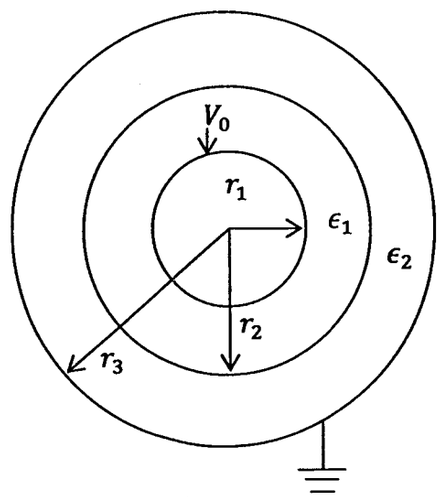
2. State whether the divergence of the following vector fields is positive, negative or zero : **5**

   a) the thermal energy flow in $J/(m^2 \cdot s)$ at any point in a freezing ice cube

   b) the current density in A/m^2 in a bus-bar carrying direct current

   c) the mass flow rate in $kg/(m^2 \cdot s)$ below the surface of water in a basin, in which the water is circulating clockwise as viewed from above.

3. A coaxial power cable having a core (conductor) radius of r_1, is filled with two concentric layers of dielectrics ϵ_1 and ϵ_2 . **10**



   a) Determine the capacitance of the cable per unit length

   b) If the conductor is at a potential V_0 and the outer shield is grounded, determine the maximum electric field in each dielectric from the following data :

   $V_0 = 1200\,V,\ \epsilon_{r_1} = 1.5, \epsilon_{r_2} = 4.5$ and $r_3 = 2r_2 = 4r_1 = 4\ cm.$

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4. A sphere of radius a and relative permittivity ϵ_r is centred on the origin of a spherical coordinate system and contains a uniform volume free charge distribution $\rho_v\ C/m^3$. Determine $\vec{D}$, $\vec{E}$, $\vec{P}$ and V everywhere and the surface and volume bound charge densities. **10**

5. A solid cylindrical nonmagnetic conductor of circular cross section has a radius of 5 mm. The conductor is inhomogenous, i.e. its conductivity varies with radial distance from axis of the conductor. The conductor is 20 m long and there is a potential dfference of 0.1V dc between its two ends. Within the conductor, $\vec{H} = 10^5\,\rho^2\,\hat{a}_\phi$ A/m, where ρ is the cylindrical coordinate. **10**

   a) Find conductivity as a function of ρ.

   b) What is the resistance of the conductor between its two ends?

   c) Find the total magnetic flux inside the conductor.

6. A coaxial transmission line has a = 5 mm and b = 20 mm. Let its center lies on the z axis and let a dc current I flow in the $\hat{a}_z$ direction in the center conductor. The volume between the conductors contain a magnetic material for which $\mu_r = 2.5$ as well as air. Find $\vec{H}$, $\vec{B}$ and $\vec{M}$ everywhere between conductors if $H_\phi = \frac{600}{\pi}\ A/m$ at ρ = 10 mm, $\phi = \frac{\pi}{2}$ and the magnetic material is located at $a < \rho < 3a$ and $0 < \phi < \pi$ **10**

7. A rectangular coil is composed of 150 turns of a filamentary conductor. Find the mutual inductance in free space between this coil and an infinite straight filament on the z axis, if the four corners of the coil are located at **10**

   (a) (0,1,0), (0,3,0), (0,3,1) and (0,1,1)