

1. a) Explain the significance of homentropicity, homenergi-  
city, and irrotationality in the analysis of fluid flows.  
b) Show that the general solution of the equation

$$(M_{\infty}^2 - 1) \frac{\partial^2 \Phi}{\partial x^2} - \frac{\partial^2 \Phi}{\partial z^2} = 0, \quad M_{\infty} > 1$$

for the perturbation potential of a supersonic flow is given by

$$\Phi = U \Phi_1(x + z \sqrt{M_{\infty}^2 - 1}) + U \Phi_2(x - z \sqrt{M_{\infty}^2 - 1})$$

where  $\Phi_1$  and  $\Phi_2$  are arbitrary functions of the respective arguments and  $U$  is the uniform free-stream velocity

- c) Show that the slope of the stream lines is given by

$$\frac{dz}{dx} = \sqrt{M_{\infty}^2 - 1} (\Phi_1' - \Phi_2') / (1 + \Phi_1' + \Phi_2')$$

where  $\Phi_1'$  and  $\Phi_2'$  are the first derivatives of  $\Phi_1$  and  $\Phi_2$  with respect to their respective arguments.

- d) From physical arguments or otherwise, show that, for flow past a thin airfoil aligned with the  $x$ -axis,  $\Phi_1$  must be degenerate (i.e., assumes a constant value) for  $z > 0$  and  $\Phi_2$  must be degenerate for  $z < 0$ .

2. a) Consider a nearly uniform, parallel flow with the following perturbation velocity potential:

$$\Phi(x, z) \equiv 0 \quad \text{for } (x - z \sqrt{M_{\infty}^2 - 1}) < 0$$

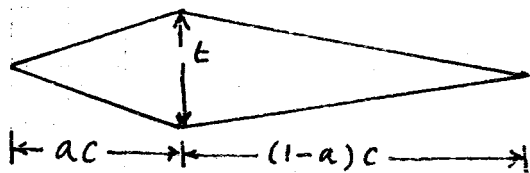
$$\Phi(x, z) = \frac{UE}{A \sqrt{M_{\infty}^2 - 1}} e^{-A(x - z \sqrt{M_{\infty}^2 - 1})} \quad \text{for } (x - z \sqrt{M_{\infty}^2 - 1}) > 0.$$

where  $A$  is a constant and  $M_{\infty}$  and  $U$  are, respectively, the free stream Mach number and velocity. Find the velocity and pressure fields corresponding to this perturbation potential

- b) Show that, when  $A$  approaches zero, the flow pattern is that corresponding to the flow from left to right over

a wall which has a sharp corner with a turning angle,  $\epsilon$ .

3. Consider the symmetric air foil shown in the figure:



i) Show that, for any arbitrary value of  $0 < a < 1$ , the lift coefficient and drag coefficient are the

same for the same values of  $M_\infty (> 1)$  and  $\alpha$  whether the flow is from left to right or from right to left.

ii) How do the moment coefficients and centres of pressure compare in the two cases?

4. a) What do you understand by subsonic and supersonic edges in the context of a wing placed in a supersonic stream? What are their implications for flow over the leading and trailing edges in a supersonic stream?

b) What are the advantages of swept wings over unswept wings in supersonic flow?

c) Ignoring the effect of skin friction, derive an expression for  $(L/D)_{\max}$  for an infinite swept back untapered wing with supersonic leading and trailing edges. Derive the expressions for the angle of attack  $\alpha'$  and lift coefficient  $C_L$  corresponding to  $(\frac{L}{D})_{\max}$ . Assume that the wing section is doubly symmetric and both  $\alpha$  and  $\phi$  are small.