Mab

Indian Institute of Technology, Kharagpur

Date..................... FN/AN Time: 2 Hrs Full Marks: 30 No. of Students: 64
Mid (Spring) Semester 2015-16 Subject Name: Switching and Finite Automata
Sub. No. MA 61002/60036/30006 Deptt: MA/ME/EE/IE/PH/CS

Instruction: Answer **all** questions. Notations used are as explained in the class.

Question 1 [$2 \times 5 = 10$ marks]

Either prove each of the following statements or show a counter example.

a) Every *finite* subset of a nonregular set is regular.

b) The expressions $P = (\mathbf{1^*0} + \mathbf{001})^*\mathbf{01}$ and $Q = (\mathbf{1^*001} + \mathbf{00101})^*$ are equivalent.

c) Let $\mathbf{R}$ denotes a regular set. Then the set consisting of all the strings in $\mathbf{R}$ that are identical to their own reverses is also a regular set.

d) Every subset of a regular set is also regular.

e) Let L_1 and L_2 be regular languages over the alphabet Σ. Define the language

$$L = \{\alpha\beta\gamma : \alpha\gamma \in L_1, \beta \in L_2\}$$

that is, L is obtained by inserting strings in L_2 inside strings in L_1. Then L is also regular.

Question 2 [$3 + 2 + 2 = 7$ marks]

a) (i) Construct an ϵ-NFA equivalent to the regular expression

$$\mathbf{0(11 + 0(00 + 1)^*)^*}$$

(ii) Convert the ϵ-NFA obtained above to a DFA.

b) Consider the following two deterministic finite automata with transition functions δ_1 and δ_2.

δ_1	a	b
$\rightarrow 1$	1	2
$*2$	2	1

δ_2	a	b
$\rightarrow 1$	2	3
2	3	1
$*3$	1	2

Use the product construction to produce deterministic automata accepting the union of the two sets accepted by these automata.

c) Give a decision algorithm to determine if the set accepted by a DFA is the set of all strings over a given alphabet.

Question 3 [$3 + 3 = 6$ marks]

a) Write an algorithm for converting a finite automaton to regular expression. Hence, or otherwise, find a regular expression on the alphabet $\{0, 1, 2\}$ for the set of strings recognized by the automaton M given in Figure 1.

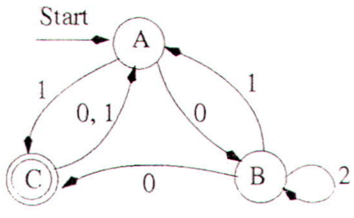

Figure 1: Automaton M

b) For the following DFA:

δ	a	b
$\rightarrow 1$	1	3
$*2$	6	3
3	5	7
$*4$	6	1
5	1	7
$*6$	2	7
7	5	3

(i) list the equivalence classes of the collapsing relation $\equiv$ defined in the class as:

$$p \equiv q \overset{\text{def}}{\Longleftrightarrow} \forall x \in \Sigma^* \ (\widehat{\delta}(p, x) \in F \Longleftrightarrow \widehat{\delta}(q, x) \in F);$$

(F being the set of accepting states of the DFA)

(ii) draw the quotient DFA obtained by collapsing equivalent states and removing inaccessible states.

Question 4 [$3 + 4 = 7$ marks]

a) Let $\Sigma = \{0, 1, \approx, \boxplus\}$, and let

$$L = \{a \approx b \boxplus c \mid a, b, c \in (0+1)^* \text{ and } a = b + c \text{ if } a, b, c \text{ are interpreted as unsigned binary integers } \}.$$

Note that the string $10 \approx 01 \boxplus 01$ is in L since $2 = 1 + 1$, but the string $10 \approx 11 \boxplus 01$ is not since $2 \neq 3 + 1$. Show that L is not regular.
(Any standard results you use should be clearly stated, but need not be proved.)

b) (i) State the Myhill-Nerode theorem.

(ii) Find the distinguishable classes of the language $L = \{a^n b a^n \mid 0 \leq n \leq 3\}$. Using Myhill-Nerode theorem design a DFA D such that $L(D) = L$.

——-The End———