AEROSPACE ENGINEERING DEPARTMENT
INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Mid-Spring Semester (2009-2010) Examination
Time: 2 Hrs.

26.02.2010 AN — AE40034, AE51024 Advanced Computational Fluid Dynamics

4^{th} Year, 5^{th} Year — No of Students: 28

Answer all questions.
Notations have their usual meaning.

1. The governing equations that represent steady shallow water flow are given by

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+g\frac{\partial h}{\partial x}=0$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+g\frac{\partial h}{\partial y}=0$$

$$\frac{\partial}{\partial x}(uh)+\frac{\partial}{\partial y}(vh)=0$$

where h represents the free surface elevation and u and v represent velocity components. Defining the state vector $\mathbf{W}$ to be $\mathbf{W}=[u,v,h]^T$, express the system of equations in a vector form and classify the system of equations.

2. The eigenvalues of 1-dimensional Euler system are $u-c$, u and $u+c$, where c is acoustic speed. The left eigenvector matrix for the system is given by

$$\mathbf{L}=\begin{bmatrix} \frac{1}{2}\left(b_1+\frac{u}{c}\right) & -\frac{1}{2}\left(b_2u+\frac{1}{c}\right) & \frac{b_2}{2} \\ 1-b_1 & b_2u & -b_2 \\ \frac{1}{2}\left(b_1-\frac{u}{c}\right) & -\frac{1}{2}\left(b_2u-\frac{1}{c}\right) & \frac{b_2}{2} \end{bmatrix}, \text{ where } b_1=b_2u^2/2;\quad b_1=(\gamma-1)/c^2.$$

Is characteristic decomposition of the Euler system possible? If not then attempt a Pfaffian characteristic (difference of characteristic variables) decomposition. Comment on the nature of solution of the Euler equation based on your decomposition.

3. Construct fully discretized and semi-discretized formulations of a system of conservation laws. Highlight the differences in the numerical flux function resulting from the two approaches. Discuss the basic principles involved in defining a numerical flux function. Considering the properties of the exact solution of a scalar hyperbolic conservation law, discuss the properties that a numerical flux function should have.

4. **Construct the numerical flux function for a scalar hyperbolic conservation law using MUSCL (monotone upstream schemes for conservation laws) on a uniform mesh. Show that the flux function obtained from the higher order MUSCL reconstruction can be expressed as the sum of a lower order flux and higher order antidiffusion term. Write down the final solution algorithm using the designed numerical flux function.**

5. **What are TVD schemes? Enumerate some of the important issues related to TVD schemes.**

An explicit numerical scheme for the inviscid Burgers equation $\left(\frac{\partial w}{\partial t}+\frac{\partial F}{\partial x}=0,\ F=\frac{w^2}{2}\right)$ is given by

$$w_i^{n+1}=w_i^n+\frac{\lambda}{2}\left[\left(\left|\alpha_{i+\frac{1}{2}}\right|-\alpha_{i+\frac{1}{2}}\right)\delta^+w_i^n-\left(\left|\alpha_{i-\frac{1}{2}}\right|+\alpha_{i+\frac{1}{2}}\right)\delta^-w_i^n\right],\quad \lambda=\frac{\Delta t}{\Delta x}.$$

Also, $\alpha_{i+\frac{1}{2}}=\begin{cases} w_i^n & \text{if } \delta^+w_i^n=0 \\ \dfrac{\delta^+F_i^n}{\delta^+w_i^n} & \text{if } \delta^+w_i^n\neq 0\end{cases}$

Is it a TVD scheme? What is the order of accuracy?