INDIAN INSTITUTE OF TECHNOLOGY

Date ..... FN / AN Time : 2 / 3 Hrs. Full Marks 100 No. of Students 90

Autumn / Spring Semester, 2009 Deptt. ..... Sub. No. 21005

2nd Yr. B.Tech. (H) / B.Arch. (H) / M.Sc. Dual degree Sub. Name Network theory

Instruction Attempt all questions - figures on the margin indicate marks

5

INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date : .....FN/AN Time 3 Hrs. Full Marks...100 No. of Students.. 90

2nd Year B.Tech (H) Dept.: E & ECE Sub. No. EC21005

End Semester (Autumn) Sub. Name : Network Theory

Instruction : i) ATTEMPT ALL THE QUESTIONS
ii) MAKE NECESSARY ASSUMPTIONS WITH JUSTIFICATIONS, IF NECESSARY.
iii) ATTEMPT ALL THE PARTS OF A QUESTION AT ONE PLACE.

1. What is the fundamental period of the signal x(t)=Cos $2\pi t$? Calculate the Fourier series coefficients of the given signal. What are the Fourier series coefficients of the same x(t) if we regard it as a periodic signal with period 3?

2+3+5

2. Consider an electronic system whose input is a linear combination of eigenfunctions $\varphi_k(t)$, each of which has a corresponding eigenvalue λ_k, i.e.

$$x(t) = \sum_{k=-\alpha}^{\alpha} c_k \phi_k(t)$$

a) Express the output of the system in terms of $\varphi_k(t)$, λ_k and c_k.
b) Consider a system characterized by the differential equation

$$y(t) = t^2 \frac{d^2x(t)}{dt^2} + t\frac{dx(t)}{dt}$$

Is this system linear? Is it time invariant?
c) Show that the function

$$\varphi_k(t) = t^k$$

are eigenfunctions of the system in part (b) . For each $\varphi_k(t)$, determine the corresponding eigenvalue λ_k.

d) Determine the output of the system if $x(t) = 10t^{-10} + 3t + \frac{1}{2}t^4 + \pi$

2+3+5+5

3. Consider a causal LTI system with frequency response

$$H(j\omega) = \frac{1}{j\omega + 3}$$

For a particular input x(t) this system is observed to produce the output

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t)$$

Determine x(t). 5

4. The output y(t) of a causal LTI system is related to the input x(t) by the equation

$$\frac{dy(t)}{dt}+10y(t)=\int_{-\infty}^{\infty}x(\tau)z(t-\tau)d\tau-x(t)$$

where $z(t)=e^{-t}u(t)+3\delta(t)$

a) Find the frequency response H(jω) of this system.
b) Determine the impulse response of this system. 4+2

5.

a) Show that Region of convergence (ROC) of the system function of a causal system is the right half of the s-plane.
b) What extra information you obtain about the ROC of H(s) if it is a rational function of s?
c) Define a stable system.
d) What extra feature you have of the rational system function of a stable, causal LTI system?
e) A minimum-delay or minimum-phase system is a causal and stable system and its inverse system is also causal and stable. Show that all the poles and zeroes of the transfer function of a minimum phase system lies in the left half of the s-plane.
3+3+2+3+ 3

6. At t=0, a switch is closed, connecting a voltage source v=VSinωt to a series RL circuit. By the method of Laplace transformation, show that the current is given by the equation

$$i=\frac{V}{Z}Sin(\omega t-\varphi)+\frac{\omega LV}{Z^2}e^{-\frac{Rt}{L}}$$

where

$$Z=\sqrt{R^2+(\omega L)^2}$$

and

$$\phi=\tan^{-1}\frac{\omega L}{R}$$

10

7. Find the transform impedance for the RLC network shown in the figure below

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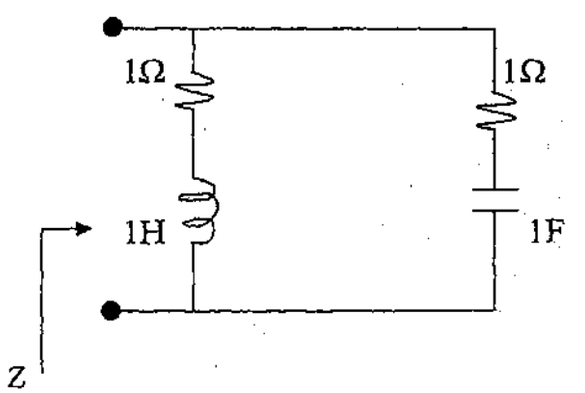

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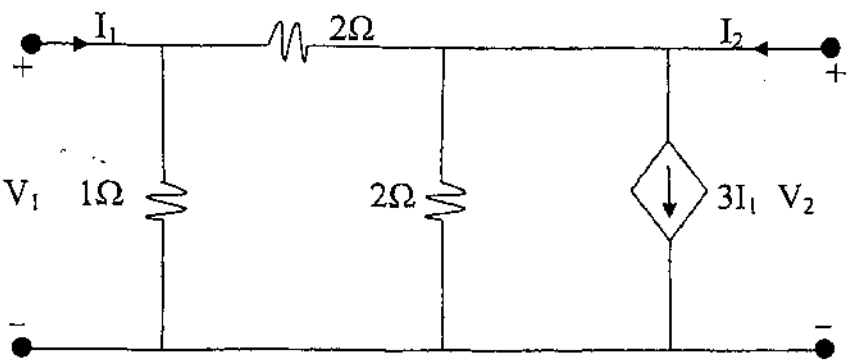
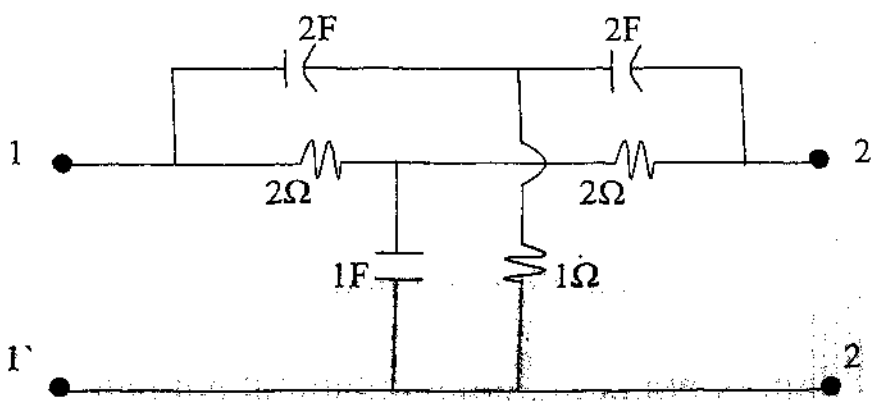
8. Find the y-parameters of the 2-port network shown below: 10



9. Find the y-parameters of the network shown below: 15



10. For the 2-port network shown below, find the voltage transfer gain and plot the Bode plot for this gain. 10