Indian Institute of Technology, Kharagpur
Mid-Autumn Semester Examination: 2018–2019

Date of Examination:....../Sep/2018 Session (FN/AN), *Duration: 2 Hrs,*
Subject. No. MA41009 *Subject Name: PROBABILITY AND STATISTICS*
Department: Mathematics *TOTAL MARKS: 30*
Specific Chart, graph paper log book etc. required.... NO...
Special Instructions (if any): .....NO..... *No. of Registered Students: 43*

ANSWER ALL THE QUESTIONS
Notations have their usual meaning.

1. Let f_n be the number of factors of $n \in \mathbb{N}$. What is the probability that f_n will be an odd number when n is chosen uniformly from $\{1, 2, \cdots, 2018\}$? [5]

2. An encryption process is using 10 distinct keys. If all the keys are arranged together in a particular order then only the signal can be completely decoded. A partially correct arrangement of all the keys will help to get some partial information. A hacker has all 10 keys with him but he is unaware of the correct arrangement. What is the probability that a random arrangement will **not** help him to get any information from the signal because of no match in the arrangement of the keys ? [5]

3. A customer can reach to the desired counter in a bank after waiting for Y min. and he leaves that counter (as well as the bank) with service after X min. of his entry into the bank. If he reached the counter after 5 min. of his entry into the bank , what is his expected time he had to spent in the bank when (X, Y) have the following joint density?

$$f(x,y) = \begin{cases} 4y(x-y)e^{-(x+y)}, & \text{if } 0 < y < x, 0 < x < \infty \\ 0, & \text{otherwies.} \end{cases}$$

[5]

4. Let $\{A_n\}$ be a sequence of independent events in probability spaces $(\Omega, \mathcal{A}, P)$ with $\sum_n P(A_n) = \infty$. Prove that $P(\ \limsup_n A_n\ ) = 1$. [5]

5. Let X and Y be random variables associates with the probability spaces $(\Omega, \mathcal{A}, P)$. Prove that XY is also a random variable. [5]

6. **Definition** : Two random variables U and V have same distribution (c.d.f) if and only if $E(g(U)) = E(g(V))$ for all bounded continuous Borel-measurable functions $g : \mathbb{R} \mapsto \mathbb{R}$.

   Suppose X, Y are independent and normally distributed with mean zero and variance σ^2. Find the distribution of $X^2 + Y^2$ **using the above definition.** [5]

****************** THE END *****************