INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date: **Time: 2 Hours** **Max. Marks: 60** **No. of Students: 25**

Spring (Mid) Semester, 2009-2010 **Department: Geology & Geophysics**
Subject No.: EX 30008/EX40002 **3rd Yr Integrated M. Sc. (Ex Geophysics) / 1st Yr M.Sc. (Geophysics)**

Subject: Geophysical Signal Processing

Instructions: *This question paper consists of two pages. All questions are compulsory. The credits are shown at the right hand margin.*

1. **(a)** Provide the unified definitions of a function, a signal and a system . You may use appropriate mathematical formulations /expressions while doing so.

(3)

(b) Classify the following signals according to whether they are (1) one- or multidimensional; (2) single or multichannel, (3) continuous time or discrete time, and (4) analog or digital (in Amplitude). Give a brief explanation for each.

(i) Closing prices of the utility stocks on the Mumbai Stock Exchange.
(ii) A color movie.
(iii) Position of the steering wheel of a car in motion relative to car's reference frame.
(iv) Position of the steering wheel of a car in motion relative to ground reference frame.
(v) Weight and height measurements of a child taken every month.

(8)

2. **(a)** Give the ***Fourier Series Expansion*** of a 3D Function g(x,y,t), where x & y are the real spatial variables in the Cartesian Coordinate system and 't' is a real time variable. Spell out the necessary and sufficient conditions the function has to satisfy to have such trigonometric expansions. Also deduce the coefficients associated with the trigonometric terms in the series. Express the function in terms of an exponential series. How do the ***Fourier Series*** get transformed to a 3rd order ***Fourier Integral***? Hence bring out the concept of ***Fourier Transform*** of this 3D signal.

(11)

(b) The sawtooth waveform in the following **Figure 1** can be expressed in the form of a Fourier Series as,

$$x(t) = \frac{2}{\pi}\left(\sin \pi t - \frac{1}{2}\sin 2\pi t + \frac{1}{3}\sin 3\pi t - \frac{1}{4}\sin 4\pi t ....\right)$$

Determine the Fourier series coefficients c_k.

(4)

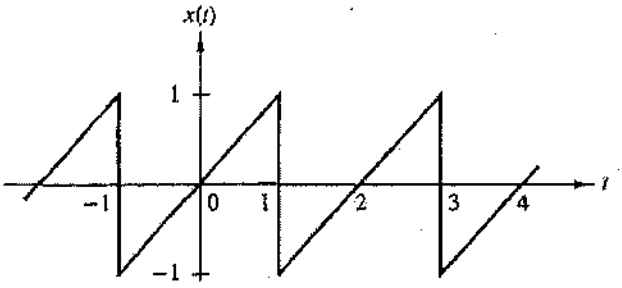

Figure 1

P.T.O.

3. **(a)** Considering a periodic signal x(t) continuous/ discrete/ transit, deduce ***Parseval's Theorem*** for its power signal. State & explain the ***Wiener-Khintchine Theorem***.

(3)

(b) Prove that the auto-correlation function and the power spectrum are each other's ***Fourier Transforms***. Deduce the ***coherence spectrum*** between two band limited signals, $f_1(t)$ and $f_2(t)$.

(4)

(c) Consider the full-wave rectified sinusoid given in **Figure 2**,

(i) Determine its spectrum $X_a(F)$.
(ii) Compute the power of the signal.
(iii) Plot the power spectral density.
(iv) Check the validity of Parseval's relation for this signal.

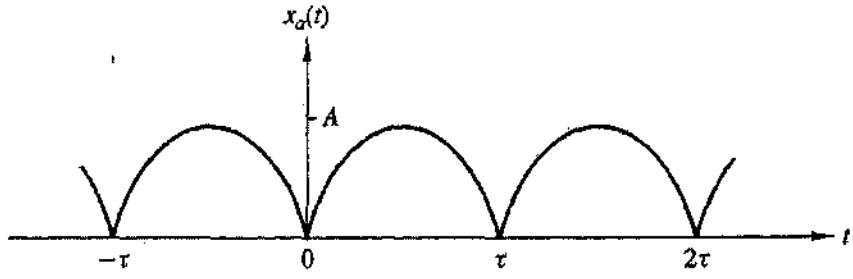

Figure 2

(6)

4. **(a)** Determine and sketch $X_R(\omega), X_I(\omega), |X(\omega)|$, and $\angle X(\omega)$ for the Fourier transform

$$X(\omega) = \frac{1}{1 - ae^{-j\omega}} \qquad -1 < a < 1$$

(b) If concepts of time and frequency are interchanged, what does the meaning of spectrum become?

(4+2=6)

5. **(a)** The Fourier Transform of a rectangle function is sin(αt)/αt , also known as a "sinc" function. In terms of α, how wide is the rectangle function?

(b) State the ***Convolution Theorem*** for a linear system. Prove that the Convolution in the time domain is equivalent to multiplication in the frequency domain.

(c) State and prove the ***Scaling Theorem*** associated with both the ***Fourier*** and ***Laplace Transforms***. Consider a single dimensional signal, say g(t) as a function of real time variable t.

(2+3+3=8)

6. Derive the intrinsic attributes of a real time Transient Signal recorded on seismic excitation of a real-earth model using both ***Fourier*** and ***Hilbert Transforms*** and hence define the amplitude, instantaneous frequency and phase of the seismic signal in both frequency and time domains. How these attributes are physically interpreted? You are required to make critical comments on these transformations and spell out the constraining conditions adhered to both the transformations.

(7)

.....End.....